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# Jan Ambjørn and Romuald A. Janik:

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Jan Ambjørn<sup> $a,\dagger,\ddagger$ </sup> and Romuald A. Janik<sup> $b,\dagger,\$$ </sup>

<sup>a</sup> The Perimeter Institute, 35 King Street North,
Waterloo, ON, N2J 2W9, Canada email: ambjorn@nbi.dk

<sup>b</sup> The Niels Bohr Institute, Blegdamsvej 17,
DK-2100 Copenhagen Ø, Denmark *email: janik@nbi.dk*

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#### Abstract

We study the *quantum* decay of D0-branes in two-dimensional 0B string theory. The quantum nature of the branes provides a natural cut-off for the closed string emission rate. We find exact quantum mechanical wavefunctions for the decaying branes and show how one can include the effects of the Fermi sea for any string coupling (Fermi energy).

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<sup>&</sup>lt;sup>‡</sup>Permanent address: The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark. email: ambjorn@nbi.dk.

 $<sup>\</sup>ensuremath{\$}^{\$}$ Permanent address: Institute of Physics, Jagellonian University, Reymonta 4, PL 30-059 Krakow, Poland. email: ufrjanik@if.uj.edu.pl

# 1 Introduction

Originally dynamical triangulations of string world-sheets were introduced as a reparametrization invariant lattice regularization. It successfully defined non-critical strings of fixed world-sheet topology in a non-perturbative way, the scaling to the continuum controlled by the lattice spacing [1, 2, 3]. The socalled matrix model description was a very convenient way of implementing the combinatorial task of summing over all abstract triangulations of the world-sheet, which automatically, via the large N expansion, arranged the world-sheets according to topology. No physical interpretation of the matrix itself was given until recently when it was suggested in the case of the c = 1matrix model that the matrix could be given the interpretation as the open string tachyon field between N unstable D-branes (N being the size of the matrix), the unstable D-branes themselves being identified as the eigenvalues of the matrices [4, 5].

This intriguing picture has passed a number of non-trivial tests (also for models with c < 1, [6, 7, 8]) and it offers the possibility for the first time to study *quantum D-branes* in *strongly* coupled string theories.

In this paper we will discuss the decay of such quantum D-branes.

The outline of this paper is as follows. In section 2 we describe the decay of the D0-brane in the classical approximation and reproduce directly from the classical motion the closed string tachyon 1-point function. In section 3 we present the exact quantum mechanical treatment of the same process, although without taking into account the effect of the Fermi sea, and show how a natural cut-off arises for the closed string emission. In section 4 we show how to exactly incorporate the effects of the string interactions (Fermi sea) in the preceeding quantum mechanical picture. We close the paper with a discussion.

# 2 Classical decay of the D0-brane

In the double scaling limit the ground state of 2d *bosonic* string theory is constructed by filling one side of the upside-down harmonic oscillator potential  $-\lambda^2/2\alpha'$  with free fermions up to a Fermi level  $-\mu$ , where zero energy is the top of the potential. The string coupling  $g_s \sim 1/\mu$  and the partition function, closed string tachyon operators, macroscopic loop operators etc have a unique perturbative expansion in  $g_s$  which can be obtained from the exactly solvable quantum mechanics of the upside-down harmonic oscillator. As first pointed out in [9] the situation is unclear if we move to strong string couplings, i.e. small  $\mu$  near the top of the potential. Clearly tunneling (so-called "non-perturbative" effects) between the two sides is no longer exponentially suppressed, but there is no "first principle" in the bosonic case telling us how to relate the two sides.

Fortunately in the case of two-dimensional 0B and 0A superstrings, as realized in [10, 11], this ambiguity is lifted and both sides are filled up to the same level.

In the following we will perform calculations within the 0B model<sup>3</sup> defined by

$$\int dT \, e^{-\int dt \left\{ \frac{1}{2} (DT)^2 + V(T) \right\}} \tag{1}$$

where the nondynamical gauge field in the covariant derivative just restricts the path integral to singlet states and so the standard free fermions give a complete description of all the degrees of freedom.

Further, the quantum states of a D0-brane are precisely the quantum states of the Hamiltonian of the inverted harmonic potential except that the spectrum starts at  $-\mu$  and the model provides us with a complete description of the dynamics of a single D0-brane [13].

The potential for the 0B matrix model differs from the pure bosonic potential by a factor of two:

$$V = -\frac{1}{4\alpha'}\lambda^2 \tag{2}$$

As realized in [4] eigenvalues may be identified with (unstable) D0-branes. Consequently time dependent solutions are of interest since they may be identified with decay processes. A solution of the classical equations of motion is

$$\lambda(t) = \sqrt{4\mu\alpha'} \sin \pi \tilde{\lambda} \cosh \frac{t}{\sqrt{2\alpha'}},\tag{3}$$

where  $\lambda$  should not be confused with the eigenvalue  $\lambda$ . Its energy relative to the Fermi surface  $E_F = -\mu$  is  $\mu \cos^2 \pi \tilde{\lambda}$ . This solution is the matrix model analog of Sen's 'rolling tachyon' [14] and it will emit closed strings tachyons, the rate dictated by its contribution to the on-shell closed string 1-point

<sup>&</sup>lt;sup>3</sup>However, all results derived here away from the strong coupling region ( $g_s$  large) are qualitatively correct also for the 2d bosonic string (the c = 1 matrix model).

function. This in turn can be computed since the *closed* string tachyon (in the NS-NS sector) can be defined [10] in the 0B matrix model  $as^4$ 

$$T_{NS-NS}(E) \sim \lim_{l \to 0} \left( \log \text{ factor} \right) \cdot \int dt \, e^{iEt} \left\langle \operatorname{tr} e^{-il\Phi^2(t)} \right\rangle \tag{4}$$

where the so-called leg-factor is

leg factor = 
$$\frac{(2il)^{iE}\sqrt{\alpha'/2}}{\Gamma(-iE\sqrt{\alpha'/2})}$$
, (5)

and the expectation value is with respect to the (double scaling limit) matrix integral:

$$\left\langle \operatorname{tr} e^{-il\Phi^2(t)} \right\rangle = \int d\lambda \ \rho_{ds}(\lambda, t) \ e^{-il\lambda^2},$$
 (6)

 $\rho_{ds}(\lambda, t)$  being the appropriate double scaling limit of the eigenvalues.

Let us compute the contribution of the classical motion  $\lambda(t)$  of the eigenvalue (3) to the 1-point function of the NS-NS closed string tachyon. The contribution of such a classical eigenvalue to  $\rho_{ds}(\lambda, t)$  is simply  $\delta(\lambda - \lambda(t))$ . Thus we have to evaluate the integral

$$\int dt \, e^{iEt} \, e^{-il4\mu\alpha' \, \sin^2\pi\tilde{\lambda} \, \cosh^2\frac{t}{\sqrt{2\alpha'}}} \tag{7}$$

in the limit  $l \to 0$ . The non-trivial small l behavior comes from the large t region of the integral and the following change of integration variable

$$u = le\sqrt{\frac{2}{\alpha'}t} \tag{8}$$

leads to an integral

$$\sqrt{\frac{\alpha'}{2}} l^{-iE\sqrt{\frac{\alpha'}{2}}} \int_0^\infty \frac{du}{u} e^{-i\mu\alpha' u \sin^2 \pi \tilde{\lambda}} u^{iE\sqrt{\frac{\alpha'}{2}}}.$$
(9)

Performing the integral, setting  $\alpha' = 2$  and inserting the leg-factor (5) we obtain the correct answer:

$$e^{-iE\log\sin^2\pi\tilde{\lambda}}\frac{\Gamma(iE)}{\Gamma(-iE)}\mu^{-iE}.$$
(10)

Analogous calculations here and below, can of course be easily done also for the closed string tachyons in the R-R sector.

<sup>&</sup>lt;sup>4</sup>In (4) we have made a rotation  $l \rightarrow il$ , in accordance with the discussion in [15], sec. 11.

# 3 'Quantum' decay - D-brane wave packets

As already mentioned the quantum theory of a D0-brane in the c = 1 theory is completely described as a free fermion with the Hamiltonian

$$H = -\frac{1}{2}\frac{d^2}{d\lambda^2} - \frac{1}{2}\kappa^2\lambda^2 \tag{11}$$

where  $\kappa = 1/\sqrt{2\alpha'}$ . Therefore the classical picture of D0-brane decay is only approximate, limited by the uncertainty principle. In the quantum theory we are forced to consider wave packets instead of localized eigenvalues following a classical trajectory with both a definite position and momentum. The only non-trivial aspect is the fermionic nature of the D0-branes, which gives an indirect interaction with the Fermi-sea background and thus reflects the nonzero value of the string coupling.

Let  $\psi(\lambda, t)$  be a normalized solution to the Schrödinger equation for Hand let us assume it has no overlap with the eigenfunctions of H with energy  $E < E_F$ . The contribution of this quantum state to the density  $\rho_{ds}(\lambda, t)$  will be  $|\psi(\lambda, t)|^2$  and thus the corresponding contribution to (6), when inserted in (4), leads to the following integral:

$$T_{NS-NS}^{quantum}(E) = \lim_{l \to 0} (\text{leg factor}) \cdot \int dt \, e^{iEt} \int d\lambda e^{-il\lambda^2} |\psi(\lambda, t)|^2.$$
(12)

For an arbitrary solution to the Schrödinger equation we can always write

$$\psi(\lambda, t) = \sum_{E} c_E \psi_E(\lambda, t).$$
(13)

If this expansion of  $\psi$  contains  $E < E_F$  they have to be cut away and the wave function re-normalized (see section 4 below).

Let us concentrate on the simplest wave packets, Gaussian wave packets, and ignore at first the problem of overlap with the Fermi sea. We will address it in the next section. First a few general observations: (1) for a quadratic potential the expectation values  $\langle \lambda(t) \rangle$  will always follow a classical orbit as follows from Ehrenfest's theorem. (2) A wave packet which is Gaussian at some time t will remain Gaussian. This follows because the propagator  $G(\lambda, \lambda'; t)$  corresponding to H is Gaussian in  $\lambda, \lambda'$ . (3) Since the peak will then coincide with the expectation value of  $\lambda$  we know that for an initial wave packet of the form

$$\psi(\lambda,0) = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}} e^{-a(\lambda-\lambda_0)^2 + ip_0\lambda}$$
(14)

we have

$$|\psi(\lambda,t)|^2 = \left(\frac{2a(t)}{\pi}\right)^{\frac{1}{2}} e^{-2a(t)(\lambda-\lambda(t))^2}$$
(15)

where  $\lambda(t)$  is just the classical orbit corresponding to initial values of  $\lambda_0, p_0$ and a(t) can be calculated to be<sup>5</sup>

$$a(t) = \frac{a}{\Delta(t)}, \quad \Delta(t) = \cosh^2 \kappa t + \frac{4a^2}{\kappa^2} \sinh^2 \kappa t$$
 (18)

Consequently the temporal evolution of wave packets is essentially dictated by the classical energy<sup>6</sup>  $H_{cl}(\lambda_0, p_0) = \frac{1}{2}(p_0^2 - \kappa^2 \lambda_0^2)$  since the peak just follows the classical orbit and the wave packet never splits in two, as a generic wave packet would do in the inverted harmonic potential. However, due to the very rapid spread of the wave one still has non-zero transmission though the potential barrier even if  $H_{cl} < 0$ . For a given initial wave packet we see that a true classical picture of a localized wave function is only valid for times times  $\kappa t < \log a$  or  $t < \sqrt{\alpha'} \log a$ .

Let us for simplicity consider a wave packet where  $p_0 = 0$  and  $\lambda_0 > 0$ . This is the wave packet version of Sen's rolling tachyon and  $\lambda_0$  will now be related to  $\mu$  as follows:

$$\lambda_0^2 = 4\mu\alpha' \sin^2 \pi \tilde{\lambda}.$$
 (19)

We can now calculate the closed string tachyon one-point function (12). The integral over the eigenvalues gives in this case

$$\frac{1}{\sqrt{1+i\frac{\Delta}{2a}l}} \exp\left(-il\frac{\lambda_0^2\cosh^2\kappa t}{1+i\frac{\Delta}{2a}l}\right)$$
(20)

<sup>5</sup>It can easily be read off from the propagator

$$G(\lambda,\lambda';t) = \left[\frac{\kappa}{2i\pi\sinh\kappa t}\right]^{1/2} \exp\left\{\frac{i\kappa}{\sinh\kappa t}\left[(\lambda^2 + {\lambda'}^2)\cosh\kappa t - 2\lambda\lambda'\right]\right\},\qquad(16)$$

$$\psi(\lambda,t) = \int d\lambda' \ G(\lambda,\lambda';t)\psi(\lambda',0). \tag{17}$$

<sup>6</sup>The actual quantum energy of the wave packet is  $\langle \psi | H | \psi \rangle = H_{cl}(\lambda_0, p_0) + \frac{a}{2} - \frac{\kappa^2}{8a}$ .

It might seem as if one could neglect the contribution  $l\Delta/2a$  from the spreading of the wave packet since we take the  $l \to 0$  limit. However since  $\Delta$  grows exponentially in time this contribution is of the same order as the classical piece. Therefore the classical result (10) will get modified. Making the change of variable (8) we have to perform now the integral

$$\sqrt{\alpha'/2} l^{-iE} \sqrt{\alpha'/2} \int_0^\infty \frac{du}{u} \, u^{iE} \sqrt{\alpha'/2} \cdot \frac{e^{-i\frac{\lambda_0^2}{4} \frac{u}{1+iCu}}}{\sqrt{1+iCu}} \tag{21}$$

where

$$C = \frac{1}{4} \left( \frac{1}{2a} + \frac{2a}{\kappa^2} \right) = \frac{1}{2} \sqrt{\alpha'/2} \left( \frac{1}{4a\sqrt{\alpha'/2}} + 4a\sqrt{\alpha'/2} \right).$$
(22)

This integral can be performed by a further change of variable  $v = \frac{iCu}{1+iCu}$ which leads to an integral of the form (with  $\alpha' = 2$ )

$$(iC)^{-iE} \int_0^1 dv v^{iE-1} (1-v)^{-iE-\frac{1}{2}} e^{-\frac{\lambda_0^2}{4C} \cdot v}$$
(23)

The final result, after including the leg-factors, is

$$quantum = \frac{\Gamma(iE)}{\Gamma(-iE)} \cdot \left(\frac{C}{2}\right)^{-iE} \Gamma\left(\frac{1}{2} - iE\right) \frac{1}{\sqrt{\pi}} F_1\left(iE, \frac{1}{2}; -\frac{\lambda_0^2}{4C}\right) (24)$$
$$classical = \frac{\Gamma(iE)}{\Gamma(-iE)} \cdot \left(\frac{\lambda_0^2}{8}\right)^{-iE}$$
(25)

where for comparison we have also written the classical result and where the relation between  $\lambda_0$  and  $\mu$  is as in (19). Note that this appearance of  $\mu$  (equivalently  $g_s$ ) here is purely 'kinematical' and enters only through the initial condition  $\lambda_0$ . The appearance of  $\mu$  due to string interactions (influence of Fermi sea) will be treated in the next section.

Let us first discuss the case when  $\lambda_0 \gg 0$ .

First note that from the asymptotics  ${}_1F_1(a,b;-x) \sim x^{-a}\Gamma(b)/\Gamma(b-a)$ we see that the quantum result for the tachyon 1-point function leads to the classical one in the limit  $\mu \to \infty$   $(g_s \to 0)$  while keeping the energies E fixed.

However for any large but finite  $\mu$  the behavior of both formulas is quite different. The classical result is a pure phase factor which for large E behaves as:

$$T_{NS-NS}^{classical}(E) \sim -i(\ldots)^{-iE} \cdot e^{-2iE} e^{2iE\log E} + \mathcal{O}(1/E)$$
(26)

Thus the number of emitted particles,  $N = \int \frac{dE}{E} |T_{NS-NS}(E)|^2$ , diverges logarithmically and the expectation value of the emitted energy diverges linearly [5] as in the similar calculation in 26 dimensions [16]. It was suggested that a cut-off of order  $1/g_s$  should be put into these calculations but in our case of quantum D0-branes we see that such a cut-off arises naturally. Indeed, denoting  $M = \lambda_0^2/(4C)$  we have the large E asymptotics

$$_{1}F_{1}\left(iE,\frac{1}{2};-M\right) \sim e^{-\frac{M}{2}}\cosh\left(2\sqrt{M}\sqrt{\frac{1}{4}-iE}\right)$$
 (27)

$$\Gamma(1/2 - iE) \sim \sqrt{2\pi} e^{-\frac{\pi}{2}E} e^{iE - iE\log E}$$
 (28)

We see that the answer is cut off when  $\pi E/2 \gg \sqrt{2ME}$ , or, dropping constants, when  $E \gg M$ . Thus the energy emitted is finite, the regularization provided by the quantum mechanical nature of the D0-brane as indeed conjectured in [5]. If the classical orbit has a turning point away from zero ( $\lambda_0 \gg 0$ ) then the cut-off is (with  $\alpha'$  reinserted and  $\mu \sqrt{\alpha'} \sim 1/g_s$ )

$$\sqrt{\alpha'}E_{cutoff} \sim \frac{1}{g_s} \frac{1}{ca\sqrt{\alpha'} + (1/ca\sqrt{\alpha'})} \sim \frac{1}{g_s},\tag{29}$$

where the constant c is of order 1 and the last ~ is valid when the location of the wave packet at t = 0 is of the order of  $\sqrt{\alpha'}$ . Anyway,  $1/g_s$  will always serve as a upper cut-off as long as the  $\lambda_0 \gg 0$ .

Let us now turn to the opposite case. When the turning point of the classical orbit is close to the maximum of the potential M goes to zero and the cut-off is now set just by (28). Then in the r.h.s. of (29)  $1/g_s$  should be replaced by 1, showing that quantum effects wash out the signature of string perturbation expansion. In particular this is the case in the "pure quantum case" where  $\lambda_0 = 0$  and the classical eigenvalue is located on the top of the potential without rolling down. This situation is not sustainable for the quantum brane and the probability of finding the eigenvalue within a distance d from the origin decreases like  $e^{-\kappa t} d\sqrt{a}$  for the Gaussian wave packet. The cut-off of the emitted energy is then just  $E_{cutoff} = \sqrt{1/\alpha'}$ .

## 4 Large $g_s$ and inclusion of the Fermi sea

Above we obtained the exact motion of the wave packet in the inverted potential. However we treated the motion independently from the other eigenvalues. This is a good approximation if the overlap with the Fermi sea is small. If we consider wave packets imitating to some degree the rolling tachyon of Sen we expect the above calculation to be reliable if  $\mu \sqrt{\alpha'} \sim 1/g_s$ is large. However, when the string coupling is large the Fermi level is close to the top and we are bound to get a significant overlap between the wave packet and the Fermi sea and it has to be taken into account.

The exact N-body wave function of the system will just be a Slater determinant with the  $\psi(\lambda, t)$  wave function derived in the previous section as one of its components. Then any observable such as the closed string tachyon 1-point function has to be computed using the Slater determinant wave function. Such a calculation would be in general quite formidable due to the needed anti-symmetrization and lots of possible subtle cancellations. This will introduce another source for the dependence on the string coupling  $g_s$  into the picture (since this is encoded in the Fermi level).

In order to bypass these complications we will construct from  $\psi(\lambda, t)$ an equivalent  $\mu$ -dependent wave function  $\psi_{eff}(\lambda, t)$  which will render antisymmetrization trivial and allow to use single particle intuitions.

To this end let us consider properly normalized Slater determinants. If all the component wave functions are orthogonal to each other then the Slater determinant will be properly normalized. This is certainly the case for the levels in the Fermi sea, but the wave packet  $\psi(\lambda, t)$  will have components also below the Fermi level. Of course these components will not contribute to the N-body wave function. So the only effect of the Fermi sea will be to truncate the original expansion of the wave packet

$$\psi(\lambda, t) = \sum_{E} c_E \ e^{-iEt} \phi_E(\lambda) \tag{30}$$

to

$$\psi_{eff}(\lambda, t) = \mathcal{N} \cdot \psi_{proj}(\lambda, t) = \mathcal{N} \sum_{E > -\mu} c_E \ e^{-iEt} \phi_E(\lambda) \tag{31}$$

where the normalization constant is

$$\mathcal{N} = \left(\sum_{E > -\mu} |c_E|^2\right)^{-\frac{1}{2}} \tag{32}$$

Note that the normalization constant  $\mathcal{N}$  will re-normalize all expectation values (average energies, 1-point functions) from the single particle case.



Figure 1: The normalization constant  $\mathcal{N}$  and the energy of the effective wave-function  $\psi_{eff}(\lambda, t)$  (corresponding to the Gaussian wave packet (35)) as a function of  $\mu$ .

The energy profiles of the 1-point functions will of course also change due to the absence in  $\psi_{proj}(\lambda, t)$  of some of the original components of  $\psi(\lambda, t)$ . Since  $\psi_{eff}(\lambda, t)$  is orthogonal to all states below the Fermi level anti-symmetrization is trivial and effectively drops out.

The projector is  $P_{E_F}(E) = \theta(E - E_F)$ , the Fourier transform of which is

$$P_{E_F}(t,s) = \frac{1}{2}\delta(t-s) + \frac{ie^{iE_F(t-s)}}{2\pi(t-s)}.$$
(33)

Thus we can write (with  $E_F = -\mu$ )

$$\psi_{proj}(\lambda,t) = \frac{1}{2}\psi(\lambda,t) + \frac{i}{2\pi} \int_0^\infty \frac{ds}{s} \left[ e^{-i\mu s} \psi(\lambda,t+s) - e^{i\mu s} \psi(\lambda,t-s) \right].$$
(34)

This integral representation is of course completely general and might be convenient whenever one actually knows the wave function  $\psi(\lambda, t)$ .

The simplest case, and the one in the family of states we have considered here which is *least* semi-classical, is

$$\psi_0(\lambda,0) = \frac{1}{(2\pi)^{\frac{1}{4}}} e^{-\frac{\lambda^2}{4}}, \qquad \psi_0(\lambda,t) = \frac{1}{(2\pi)^{\frac{1}{4}}} \frac{1}{\sqrt{\cosh\frac{t}{2}}} \frac{e^{-\frac{\lambda^2}{4}\frac{1-i\tanh\frac{t}{2}}{1+i\tanh\frac{t}{2}}}}{\sqrt{1+i\tanh\frac{t}{2}}}, \quad (35)$$

where the last expression follows from (17). With this choice of wave function both the classical energy  $H_{cl} = \frac{1}{2}(p_0^2 - \kappa^2 \lambda_0^2)$  and the quantum energy  $\langle \psi_0 | H | \psi_0 \rangle = H_{cl}(\lambda_0, p_0) + \frac{a}{2} - \frac{\kappa^2}{8a}$  are zero.



Figure 2: The wave-functions  $\psi_{eff}(\lambda, t = 0)$  (dotted points) corresponding to the Gaussian wave-packet  $\psi_0(\lambda, t = 0)$  in eq. (35) (shown as a solid line) calculated for  $\mu = 1.5, 1.0, 0.5$  and 0.

In order to extract  $\psi_{eff}(\lambda, t)$  from  $\psi_0(\lambda, t)$  one can use the general formula (34) to get after some manipulations<sup>7</sup>:

$$\psi_{proj}(x,t) = \psi(x,t) - \frac{i}{\sqrt{2\pi^{\frac{5}{4}}}} e^{-\frac{\pi}{2}\mu} e^{i\mu t} \int_{-1}^{1} \frac{(1-v)^{-\frac{3}{4}+i\mu}(1+v)^{-\frac{3}{4}-i\mu} e^{i\frac{x^2}{4}v}}{t+i\frac{\pi}{2}+\log\frac{1-v}{1+v}} dv$$
(36)

Alternatively in this particular case one can explicitly find the decomposition in energy eigenstates<sup>8</sup>:

$$\psi_0(\lambda, t) = \int dE \, c_E \psi_E(\lambda) e^{-iEt}, \qquad c_E = 2 \frac{\sqrt{\left|\frac{\Gamma(1/4 - iE)}{\Gamma(3/4 - iE)}\right|}}{(4\pi \cosh(2\pi E))^{\frac{1}{4}}} \tag{37}$$

where the coefficients fall off exponentially for large |E|,  $c_E \sim |E|^{-1/4} e^{-\pi |E|/2}$ . For large  $\mu$  the overlap with the Fermi sea will indeed be exponentially small

<sup>&</sup>lt;sup>7</sup>Write (34) as  $\int_{\mu}^{\infty} d\mu \frac{d}{d\mu} \psi_{proj}$ , interchange the  $\mu$  and s integrations and change variables from s to v.

<sup>&</sup>lt;sup>8</sup>The parity even parabolic cylinder functions  $\psi_E(\lambda)$  can be found in [12, 15], we use here the normalization from [15] and  $E = -\frac{a}{2}$  where a is the parameter used in [15].

in  $\mu$  and thus "non-perturbative" in nature. However, the "non-perturbative" nature is of the same origin as most other "non-perturbative" corrections discussed in the literature since the exponential nature of the correction comes from the Fermi-sea wave functions being in a classically forbidden region (around  $\lambda = 0$ ). The same can be said about the normalization constant  $\mathcal{N}$  from (32). Note also that the energy of this "quantum" brane will change from zero to a positive value due to the interaction with the Fermi sea.

In Fig. 1 we have shown the behavior of the normalization constant  $\mathcal{N}$  and the energy  $E_{eff}$  defined by

$$E_{eff} = \langle \psi_{eff} | H | \psi_{eff} \rangle \,. \tag{38}$$

Only for  $\sqrt{\alpha' \mu} < 1$ , i.e. for  $g_s > 1$  is there an effect which is not exponentially suppressed in  $\mu$ . This is corroborated by looking at the wave function  $\psi_{eff}(\lambda)$ itself. However, for small  $\mu$  we see quite a large change in the wave function due to the interaction with the Fermi sea, as shown in Fig. 2.

## 5 Discussion

This work was motivated by the calculations of closed string emission in the time-dependent rolling tachyon open string background of Sen. These calculations, in the context of 26-dimensional bosonic string theory were performed in [16] and in the context of 2d critical string theory and/or matrix models in [4, 5, 11, 10]. The observation was that the emitted energy was generically infinite. In the *concrete* calculations the open string background was viewed as purely classical<sup>9</sup>, which in the matrix model formulation translates into the statement that the eigenvalue corresponding to the D0-brane follows a classical trajectory in the inverted harmonic potential. Indeed, we saw that it was very simple to perform the closed string emission rate calculation using this classical trajectory. Here again the emitted energy is infinite, in accordance with the fact that the D0-brane is treated as a classical background object.

Since the *quantum states* of a D0-brane are in a one to one correspondence with the quantum states of the cut-off Hamiltonian in the inverted

 $<sup>^{9}\</sup>mathrm{In}$  [5] there is a discussion of using bosonization of chiral fermions to get a more complete treatment.

harmonic potential, the matrix model offers a simple way to move away from the classical situation. In this paper we derived an exact quantum description of the time-dependent decay process of the unstable D0-brane in type 0B two-dimensional superstring theory. By doing so the emitted energy indeed becomes finite, and for small  $g_s$  it is cut off at  $\sqrt{\alpha' E} = 1/g_s$ . For large  $g_s$  the cut-off is of order 1.

As often before, we are in a situation where the matrix model offers us a simple way of addressing questions which are not easily addressed in the continuum or higher dimensional theory. In particular here we could exactly treat a quantum open string background since, as discussed above, the quantum mechanics of fermions in the upside-down harmonic potential with energy levels below the Fermi level at  $E_F = -\mu$  filled should be viewed as a candidate for the continuum quantum open string field theory in 2d, describing the dynamics of D0-branes.

In order to apply the lesson from matrix models to higher dimensional critical string theories one needs to understand how to describe the quantum D-brane (quantum open string background) from a string point of view. One possibility is to use Witten's open string field theory at the quantum level. However it would be most attractive to have a direct *worldsheet* description. Presently we can understand that the classical D-brane should be associated with a boundary conformal field theory, but the quantum D-brane generalizes this situation, and there should be a suitable string-theoretical description of the quantum D-brane. The ease with which the situation is handled in the matrix model context and the fact that very sensible results emerge is a strong hint that there should exist a simple higher dimensional string description too.

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