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The Wiener Disorder Problem with Finite Horizon

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Abstract**

The Wiener disorder problem seeks to determine a stopping time which is as close as possible to the (unknown) time of 'disorder' when the drift of an observed Wiener process changes from one value to another. In this paper we present a solution of the Wiener disorder problem when the horizon is finite. The method of proof is based on reducing the initial problem to a parabolic free-boundary problem where the continuation region is determined by a continuous curved boundary. By means of the change-of-variable formula containing the local time of a diffusion process on curves we show that the optimal boundary can be characterized as a unique solution of the nonlinear integral equation.

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Key words and phrases: Disorder (quickest detection, change-point, disruption, disharmony) problem, Wiener process, optimal stopping, finite horizon, parabolic free-boundary problem, a nonlinear Volterra integral equation of the second kind, curved boundary, a change-of-variable formula with local time on curves.

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