

The Russian Option: Finite Horizon

GORAN PESKIR*

Abstract

We show that the optimal stopping boundary for the Russian option with finite horizon can be characterized as a unique solution of the nonlinear integral equation arising from the early exercise premium representation (an explicit formula for the arbitrage-free price in terms of the optimal stopping boundary having a clear economic interpretation). The results obtained stand in a complete parallel with the best known results on the American put option with finite horizon. The key argument in the proof relies upon a local time-space formula.

*MaPhySto – A Network in Mathematical Physics and Stochastics, funded by The Danish National Research Foundation.

Mathematics Subject Classification 2000. Primary 91B28, 35R35, 45G10. Secondary 60G40, 60J60.

Key words and phrases: Russian option, finite horizon, no-arbitrage price, optimal stopping, smooth-fit, geometric Brownian motion, free-boundary problem, nonlinear integral equation, local time-space calculus, curved boundary.

REFERENCES

- [1] CARR, P. JARROW, R. and MYNENI, R. (1992). Alternative characterizations of American put options. *Math. Finance* 2 (78-106).
- [2] JACKA, S. D. (1991). Optimal stopping and the American put. *Math. Finance* 1 (1-14).
- [3] KARATZAS, I. and SHREVE, S. E. (1998). *Methods of Mathematical Finance*. Springer-Verlag.
- [4] KIM, I. J. (1990). The analytic valuation of American options. *Rev. Financial Stud.* 3 (547-572).
- [5] KOLODNER, I. I. (1956). Free boundary problem for the heat equation with applications to problems of change of phase I. General method of solution. *Comm. Pure Appl. Math.* 9 (1-31).
- [6] PESKIR, G. (1997). Optimal stopping of the maximum process: The maximality principle. *Research Report No. 377, Dept. Theoret. Statist. Aarhus* (30 pp). *Ann. Probab.* Vol. 26, No. 4, 1998, (1614-1640).
- [7] PESKIR, G. (2002). On the American option problem. *Research Report No. 431, Dept. Theoret. Statist. Aarhus*, (13 pp).
- [8] SHEPP, L. A. and SHIRYAEV, A. N. (1993). The Russian option: Reduced regret. *Ann. Appl. Probab.* 3 (631-640).
- [9] SHEPP, L. A. and SHIRYAEV, A. N. (1994). A new look at the Russian option. *Theory Probab. Appl.* 39 (103-119).
- [10] SHIRYAEV, A. N. (1999). *Essentials of Stochastic Finance (Facts, Models, Theory)*. World Scientific.

Goran Peskir
Department of Mathematical Sciences
University of Aarhus, Denmark
Ny Munkegade, DK-8000 Aarhus
home.imf.au.dk/goran
goran@imf.au.dk