Report on
Workshop on Geometric Scattering
MaPhySto, University of Aarhus
November 5–7, 1998

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Report on the Workshop

As part of the activities of MaPhySto a workshop on geometric scattering was organized at University of Aarhus, November 5–7, 1998. The workshop was narrowly focused on geometric scattering, and in particular the use of geometric scattering in understanding the structure of the scattering operator for the quantum mechanical many-body problem. A number of other questions were also discussed in detail, including the resonances and various geometric questions.

Below is first the program of the workshop, and then a collection of previews, abstracts, and reports on the lectures, with extensive references. The last section contains the list of participants.

The organizers would like to thank the lecturers for their efforts, and the participants for their active participation in the workshop.

Aarhus, December 1, 1998
Arne Jensen
Erik Skibsted

Program

Thursday November 5 (Room D1)

10.00-11.00: R. Melrose: Euclidean space and the scattering structure asymptotics
11.00-11.30: Tea/Coffee
11.30-12.30: A. Vasy: Introduction to many-body scattering
12:30-14.00: Lunch
14.00-15.00: M. Zworski: Scattering theory on manifolds with cylindrical ends: scattering matrix, resonances
15.00-15.30: Tea/Coffee
15.30-16.30: G. Salomonsen: η-Invariants for manifolds with corners

Friday November 6 (Room D1)

10.00-11.00: M. Zworski: Proof of the trace formula and of scattering asymptotics
11.00-11.30: Tea/Coffee
11.30-12.30: R. Melrose: Pseudodifferential operators and propagation of singularities
12:30-14.00: Lunch
14.00-15.00: A. Vasy: The scattering calculus
15.00-15.30: Tea/Coffee
15.30-16.30: P. Perry: Isoscattering Schottky manifolds

Saturday November 7 (Room D3)

10.00-11.00: A. Vasy: Propagation of singularities and structure of S-matrices
11.00-11.30: Tea/Coffee
11.30-12.30: M. Zworski: Pair correlation problem for phase shifts
12:30-14.00: Lunch
14.00-15.00: R. Melrose: Scattering for other classes of metrics
15.00-15.30: Tea/Coffee
15.30-16.30: Open problems – discussion

Report on lectures at Aarhus by R. Melrose

In these three lectures at the meeting at Aarhus I tried to describe the transition from 'traditional' scattering theory to geometric scattering theory and to give an idea of part, at least, of the setting for the latter and some of the methods it utilizes.

The first lecture dealt with the traditional Euclidean scattering and geometric constructions related to it. The basic domain of scattering theory is the treatment of the flat Laplacian on Euclidean space and more especially perturbations of it. Three, or more, currents may be distinguished in this enormous subject associated, respectively, with stationary methods and the two distict time-dependent methods exploiting the Schrödinger equation and the wave equation. Although personally I have been strongly influenced by the third of these approaches, especially as described in the work of Lax and Phillips [3], I spoke mostly about the stationary, or spectral, setting. Since much of the mathematical Physics literature is couched in terms of the (time-dependent) Schrödinger picture it would be particularly useful to relate this, in detail, to the microlocal discussion here. I advance my fellow-speaker András Vasy as the natural candidate to carry this out.

More explicitly, in the first lecture I discussed the parameterization of the continuous spectrum of the flat Laplacian. Using the Fourier transform this is done in terms of 'plane wave' solutions and then as a map from distributions on the 'sphere at infinity'. I then discussed at some length the radial compactification of $\mathbb{R}^n$ to a half-sphere, in which the sphere at infinity is incorporated into the space. This radial compactification naturally leads to the reinterpretation of the Euclidean metric as a metric, complete on the interior and hence singular at the boundary, on the half-sphere. This realization of Euclidean space generalizes to the notion of a *scattering metric* on an arbitrary compact manifold with boundary ([9]). At the end of the lecture I described the extension, from Euclidean space to a general scattering metric on a compact manifold with boundary, of the results on the parameterization of the spectrum ([13]).

In the second lecture I emphasized the methods involved in the generalization of results from Euclidean space, where the Fourier transform is available, to the case of a scattering metric. This included a discussion of the algebra of scattering pseudodifferential operators, starting from its realization in Euclidean space ([14]). More particularly I described the associated scattering wavefront set. This included the examination of the configuration space, obtained by blow-up techniques, and of the geometry of the scattering compactification of the tangent bundle ([9]). This lecture concluded with a brief treatment of the main analytic tool, the positive commutator method for the propagation of singularities. In fact, András Vasy gave a fuller version of this in his second lecture ([15, 16, 17, 18]).

The third lecture was devoted to an overview of the types of complete Riemann metrics which have, so far, been treated by these geometric methods. The three basic classes of metrics on a compact manifold with boundary ([10]) – cylindrical ends (b-structure, aspects of which were examined in the lectures of Maciej Zworski) ([8], [12], [1]), conformally compact (0-structure) ([4, 7, 5]) and the previously discussed scattering metrics – were introduced. The general properties they exhibit: spectral features of the Laplacian, scattering matrix, pseudodifferential algebra, configuration spaces and index formula, were compared, one to another. Then the list of 'boundary structures' was expanded to include quadratic scattering ([19]), double zero ([11]) and theta structures ([2]). The behaviour of the scattering matrix in these various cases was related to the asymptotic properties of geodesics. Finally various further extensions were mentioned, involving fibrations of the boundary ([6]), and manifolds with corners. These included the many-boundary problem as discussed in the lectures by András Vasy and iterated-conic metrics associated with stratified spaces.

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Lectures on the propagation of singularities in many-body scattering by A. Vasy

Let $\mathcal{X}$ be a (finite) family of linear subspaces X_a, $a \in I$, of $\mathbb{R}^n$, and let X^a be the orthocomplement of X_a. Let π_a and π^a denote the orthogonal projections to X_a and X^a respectively. A many-body Hamiltonian is an operator of the form

$$H = \Delta + \sum_{a \in I} (\pi^a)^* V_a; \tag{1}$$

here Δ is the positive Laplacian on $\mathbb{R}^n$ and the V_a are real-valued functions on X^a in an appropriate class. For instance, they can be polyhomogeneous symbols of order -1 which includes examples such as the Coulomb potential if its singularity at the origin is removed. Thus H is a self-adjoint operator on $L^2 = L^2(\mathbb{R}^n)$. The study of many-body scattering is an effort to understand the properties of this operator, H, in detail. We refer to [17] for the definitions of the basic objects in scattering theory; or see [23] for the definitions and the connection to the geometric point of view taken in these lectures.

Certain fundamental properties of H, especially global ones, such as its spectrum, resolvent estimates, the existence of wave operators and asymptotic completeness, are well understood [16, 17, 3, 4, 10, 5, 18, 2, 25]. Local, and hence microlocal results, are more rare. Thus, while the structure of the 2-cluster to 2-cluster S-matrix as well as that of the 2-cluster to 3-cluster S-matrix in 3-body scattering has been understood for some years now (they have smooth kernels, a *global* statement), see the works of Isozaki, Bommier and Skibsted [8, 19, 1], results involving other clusters are of more recent origin, starting with my paper [21] on the structure of the free-to-free (3-cluster to 3-cluster) S-matrix in 3-body scattering (the wave front relation is given by the broken geodesic flow on the sphere at 'infinity', a *microlocal* statement).

The purpose of these lectures is to state and explain these microlocal results, at least giving an outline of their proofs. In particular, the similarity to traditional microlocal analysis (often in singular settings) is stressed. The similarities between certain features of scattering theory and the study of hyperbolic PDE's were first emphasized by Melrose in his geometric generalization of the two-body problem [12], followed by the FIO-type construction of the Poisson operator by Melrose and Zworski in the same setting [15], though it had been known for some time that the role of the base and cotangent variables are reversed when passing from the standard pseudo-differential operator (ps.d.o.) algebra on $\mathbb{R}^n$ to ps.d.o's of interest in scattering (see especially [9, 5, 6, 26] or [7, Chapter XXX]).

In our geometrically more complicated setting we introduce a wave front set, $\mathrm{WF}_{\mathrm{Sc}}$, at spatial infinity (i.e. at the boundary $\partial\mathbb{S}^n_+$ of the radial compactification of $\mathbb{R}^n$ into a hemisphere $\mathbb{S}^n_+$), and we analyze propagation of 'singularities' (i.e. of $\mathrm{WF}_{\mathrm{Sc}}(u)$) for solutions of $(H-\lambda)u=0$, $\lambda\in\mathbb{R}$. This is closely related to the results on the structure of the S-matrices. The analogy in traditional microlocal analysis in this case is given by the wave equation on spaces with smooth boundaries and the reflection of singularities of its solutions [13, 14], [7, Chapter XXIV], transmission problems, and more generally reflected singularities of solutions of the wave equation on spaces with corners ω [11]. To see the analogy in the geometry, we need to consider the intersection C_a of $\partial\mathbb{S}^n_+$ with the closure of the image of X_a (under the compactification); V_a will be singular at C_a (it is not even continuous there) since it decays in all directions away from C_a but not at C_a (unless it vanishes identically). The role of the C_a then corresponds to that of the boundary faces (hypersurfaces or corners) of ω.

From the technical point of view, these results are based on microlocal positive commutator estimates. That is, we consider operators whose commutator with H is, to top order, positive in the part of the phase space where we wish to conclude that u is 'nice' (i.e. that the region is disjoint from $\mathrm{WF}_{\mathrm{Sc}}(u)$). Such microlocal positive commutator techniques were used, for example, in the proofs of the reflected singularities results of [13, 7] since they are very amenable to complicated geometric settings. Positive commutator results have also played a major role in many-body scattering starting with the work of Mourre [16], Perry, Sigal and Simon [17], Froese and Herbst [4], Jensen [9], Gérard, Isozaki and Skibsted [5] and Wang [24]. However, these results involved *globally* positive commutators (to 'top order', i.e. modulo compact operators in this case). Thus, in a sense these lectures combine fundamental tools of traditional microlocal analysis and many-body scattering to prove microlocal results in the latter setting.

The lecture plan more or less follows a revised version of my PhD thesis [20], but there are simplifications and improved results taken from my forthcoming manuscript [22].

1. Lecture 1: Introduce the compactified spaces, describe a geometric generalization of Euclidian many-body scattering, define the basic objects (Poisson operator, S-matrices), and state the results on the propagation of singularities of generalized eigenfunctions of many-body Hamiltonians (that is, tempered distributional solutions of $(H-\lambda)u=0$) and the structure of the S-matrices.

2. Lecture 2: Introduce the scattering calculus $\Psi^{m,l}_{\mathrm{sc}}$, the associated wave front set $\mathrm{WF}_{\mathrm{sc}}(u)$ (drawing on Richard Melrose's lectures) and its

many-body version $\Psi^{m,l}_{\mathrm{Sc}}$ and $\mathrm{WF}_{\mathrm{Sc}}(u)$. Describe the positive commutator proof for the propagation of singularities in the scattering calculus (2-body type scattering) and its analogies to traditional microlocal analysis (e.g. propagation of singularities for solutions of the wave equation). Discuss global positive commutator estimates such as the Mourre estimate.

3. Lecture 3: Prove the propagation of singularities (i.e. of $\mathrm{WF}_{\mathrm{Sc}}(u)$) for solutions of $(H-\lambda)u=0$ where H is a many-body operator satisfying certain conditions, and show how closely related results imply the corresponding structure theorems for the S-matrices.

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Lectures on scattering on manifolds with cylindrical ends by M. Zworski

Scattering on manifolds with cylindrical ends is a natural subject in geometrical analysis. It gives the simplest geometrical scattering model but it is already connected to the Atiyah-Patodi-Singer Index Theorem [3] and is a building block in the understanding of scattering on locally symmetric spaces. My lectures are intended to be elementary and focused on natural open problems.

1. Scattering theory on manifolds with cylindrical ends: scattering matrix, resonances [4],[1].

2. Proof of the trace formula and of scattering asymptotics (based on the work of Christiansen [1] and of Christiansen and myself [2]). I will also formulate a conjectural Poisson formula motivated by the recent progress in local trace formulæ for resonances – see [8] and references given.

3. Pair correlation problem for phase shifts: the Berry-Tabor conjecture for surfaces with cylindrical ends (based on the recent work of Zelditch and myself [7]). In this lecture I would also like to motivate the study of "pair correlations" in spectral and scattering theories – see [5],[6].

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Lecture on isoscattering Schottky manifolds by P. Perry

This lecture concerns joint work with Ruth Gornet and Robert Brooks.

The goal of this lecture is to explore the spectral geometry of the scattering operator by exhibiting examples of infinite volume hyperbolic three-manifolds that are 'isoscattering' in a sense we will make precise, but have distinct geometries. To do so we will work with convex co-compact hyperbolic manifolds associated to Schottky groups of hyperbolic isometries. We will use Sunada's method which has produced a large number of examples of isospectral compact manifolds, and has also been used to construct isoscattering Riemann surfaces. To our knowledge, the examples we will describe are the first examples to be constructed involving hyperbolic manifolds in three dimensions.

Our main result is:

Theorem *There exist hyperbolic three-manifolds manifolds X_1 and X_2 such that*

(i) *X_1 and X_2 have the same scattering poles*

(ii) *X_1 and X_2 have conformally equivalent boundaries*

(iii) *X_1 and X_2 are not isometric*

See references [1, 3, 8] for other constructions of hyperbolic manifolds with the same scattering poles, references [4, 5, 6] for background on scattering theory for hyperbolic manifolds, and reference [7] for the Sunada method.

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Lecture on eta-invariants for manifolds with corners by G. Salomonsen

The η-invariant of a Dirac operator on an odd dimensional manifold occurs naturally as a correction term in the Atiyah-Patodi-Singer index theorem [APS] for manifolds with boundary. It is a global spectral invariant of a the Dirac operator measuring the spectral assymmetry. The sign of the η-invariant depends on the orientation, such that the η-invariants cancel each other in gluing formulas for indices.

In the same way η-invariants for manifolds with boundary occur naturally when one considers index theory for manifolds with corners of codimension 2. In [Mü1] η-invariants for manifolds with boundary are studied and in [Mü2] and [HMM] they come out as correction terms in L^2-index formulas for manifolds with corners of codimension 2. It is worth noticing that gluing formulas for manifolds with corners do not give simple additivity of the indices. In contrast, since [Wa] it has been known that additional terms, Maslov indices, arise in such gluing formulas. These terms are intimately related to splitting formulas for η-invariants [Bu], where η-invariants of closed manifolds are split into η-invariants of manifolds with boundary. Notice that η-invariants of manifolds with boundary depend on the (Atiyah-Patodi-Singer type) boundary conditions imposed. This is not a big problem since there is a canonical choice of boundary conditions, and a variation formula for the η-invariant under change of boundary conditions [LW], [Mü1], [Wo].

Compared to the case of η-invariants for manifolds with boundary, there are considerably more ways to define η-invariants of manifolds with corners. In this talk I give a definition of η-invariants for manifolds with corners of codimension 2, which has been designed such that a splitting formula generalizing the splitting formula for manifolds with boundary can be proved, so that we get gluing formulas for indices of manifolds with corners of codimension 3 similar to the ones which are currently known for manifolds with corners of codimension 2. A sketch of the proof of this splitting formula, building on my own approach to index theory for manifolds with corners, is given.

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