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A non-perturbative Lorentzian path integral for gravity

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Abstract

A well-defined regularized path integral for Lorentzian quantum gravity in three and four dimensions is constructed, given in terms of a sum over dynamically triangulated causal space-times. Each Lorentzian geometry and its associated action have a unique Wick rotation to the Euclidean sector. All space-time histories possess a distinguished notion of a discrete proper time and, for finite lattice volume, the associated transfer matrix is self-adjoint, bounded and strictly positive. The degenerate geometric phases found previously in dynamically triangulated Euclidean gravity are not present. The phase structure of the new Lorentzian quantum gravity model can be readily investigated by both analytic and numerical methods.

In spite of numerous efforts, we still have not found a consistent theory of quantum gravity which would enable us to describe local quantum phenomena in the presence of strong gravitational fields. Moreover, the entanglement of technical problems with more fundamental issues concerning the structure of such a theory often makes it difficult to pinpoint why any particular quantization program has not been successful. There seems to be a broad consensus that a correct non-perturbative treatment should involve in an essential way the full "space of all metrics" (as opposed to linearized perturbations around flat space) and the diffeomorphism group, i.e. the invariance group of the classical gravitational action,

$$S[g_{\mu\nu}] = \frac{k}{2} \int d^d x \, \sqrt{-\det g} \, (R - 2\Lambda), \qquad d = 4, \tag{1}$$

with $k^{-1} = 8\pi G_{\mathrm{Newton}}$ and the cosmological constant Λ.

One approach that does not rely on the existence of supersymmetry tries to define the theory by means of a non-perturbative path integral. The aim is not to evaluate this in a stationary-phase approximation, but as a genuine "sum over all geometries". Since even the pure gravity theory in spite of its large invariance group possesses local field degrees of freedom, such a sum must be regularized to make it meaningful. The two most popular approaches, quantum Regge calculus and dynamical triangulations [1], both employ simplicial discretizations of space-time, on which then the behaviour of metric and matter fields is studied. One drawback of these (mainly numerical) investigations is that so far they have been conducted only for *Euclidean* space-time metrics $g^{eucl}_{\mu\nu}$ instead of the physical, Lorentzian metrics. This is motivated by the analogy with non-perturbative Euclidean (lattice) field theories on a fixed, flat background, whose results can under suitable conditions be "Wick-rotated" to their Minkowskian counterparts. The amplitudes $\exp iS$ are substituted in the Euclidean path integral by the real weights $\exp(-S^{eucl})$ for each configuration. *Real* weight factors are mandatory for the applicability of standard Monte Carlo techniques and, more generally, for the convergence of the state sums.

Unfortunately, it is not at all clear how to relate path integrals over Euclidean geometries to those over Lorentzian ones. This has to do with the fact that in a generally covariant field theory like gravity the metric is a dynamical quantity, and not part of the fixed background structure. For general metric configurations, there is no preferred notion of "time", and hence no obvious prescription of how to Wick-rotate.

One might worry that with a discretization of space-time the diffeomorphism invariance of the continuum theory is irretrievably lost. However, the example of two-dimensional Euclidean gravity theories provides evidence that this is not necessarily so. There, one can choose a conformal gauge in the continuum formulation, and obtain an effective action by a Faddeev-Popov construction. Physical quantities computed in this approach coincide with those computed using an intermediate discretization in the form of so-called dynamical triangulations. In this latter approach one approximates the sum over all metrics modulo diffeomorphisms by a sum over all

possible equilateral triangulations of a given topological manifold. Local geometric degrees of freedom are given by the geodesic lengths (all equal to some constant a) of the triangle edges and by deficit angles around vertices. Since different triangulations correspond to different geometries, the set-up has no residual gauge invariance, and it is straightforward to define (regularized versions of) diffeomorphism-invariant correlation functions. In the continuum limit, as the diffeomorphism-invariant cutoff a is taken to zero, the sum over triangulations gives rise to an effective measure on the space of geometries (whose explicit functional form is not known).

This makes the regularization by dynamical triangulations, which is also applicable in higher dimensions, an appealing method for investigating quantum gravity theories. It is a further advantage that the formalism is amenable to numerical simulations, which have been conducted extensively in dimensions 2, 3 and 4. Alas, in $d = 3, 4$, and for Euclidean signature, an interesting continuum limit has not been found. This seems to be related to the dominance of degenerate geometries. At small bare gravitational coupling $G_{\rm Newton}$, the dominant configurations are branched polymers (or "stacked spheres") with Hausdorff dimension $d_H = 2$, whereas at large $G_{\rm Newton}$ the geometries "condense" around one or several singular links or vertices with a very high coordination number, resulting in a very large d_H. These extreme geometric phases and the first-order phase transition separating them are qualitatively well described by mean field calculations [2, 3]. Another unsatisfactory aspect of the Euclidean model (as well as other discretized models of gravity) is our inability to rotate back to Lorentzian space-time.

In order to tackle these problems, two of us have recently constructed a Lorentzian version of the dynamically triangulated gravitational path integral in *two* dimensions [4]. The basic building blocks are triangles with one space-like and two time-like edges. The individual Lorentzian geometries are glued together from such triangles in a way that satisfies certain causality requirements. The model turns out to be exactly soluble and its associated continuum theory lies in a new universality class of 2d gravity models distinct from the usual Euclidean Liouville gravity. One central lesson from the two-dimensional example is that the causality conditions imposed on the Lorentzian model act as a "regulator" for the geometry. Most importantly, they suppress changes in the spatial topology, that is, branching of baby universes is not allowed. As a result, the effective quantum geometry in the Lorentzian case is smoother and in some senses better behaved: a) in spite of large fluctuations of the geometry, its Hausdorff dimension has the canonical value $d_H = 2$, unlike in Euclidean gravity, which has a fractal dimension $d_H = 4$; b) in spite of a strong interaction between matter and gravity when the system is coupled to Ising spins, the combined system remains consistent even beyond the $c = 1$ barrier, unlike what happens in Euclidean gravity [5, 6].

Motivated by these results, we have constructed a discretized Lorentzian path integral for gravity in 3 and 4 space-time dimensions. Unlike in two dimensions, the action is no longer trivial, and the Wick rotation problem must be solved. Also the geometries themselves are more involved, and the geometry of the $(d-1)$-dimensional

spatial slices is no longer described by just a single variable. We have succeeded in constructing a model with the following properties:

(a) Lorentzian space-time geometries (histories) are obtained by causally gluing sets of Lorentzian building blocks, i.e. d-dimensional simplices with simple length assignments;

(b) all histories have a preferred discrete notion of proper time t; t counts the number of evolution steps of a transfer matrix between adjacent spatial slices, the latter given by $(d-1)$-dimensional triangulations of equilateral Euclidean simplices;

(c) for a fixed space-time volume N_d (= number of simplices of top-dimension), both the Euclidean and the Lorentzian discretized gravity actions are bounded from above and below;

(d) the number of possible triangulations is exponentially bounded as a function of the space-time lattice volume;

(e) each Lorentzian discrete geometry can be "Wick-rotated" to a Euclidean one, defined on the same (topological) triangulation;

(f) at the level of the discretized action, the "Wick rotation" is achieved by an analytical continuation in the dimensionless ratio $\alpha = -l^2_{\rm time}/l^2_{\rm space}$ of the squared time- and space-like link length; for $\alpha = -1$ one obtains the usual Euclidean action of dynamically triangulated gravity;

(g) for finite lattice volume, the discrete transfer matrix is a self-adjoint, bounded operator which is strictly positive, i.e. all of its eigenvalues are larger than zero;

(h) the extreme phases of degenerate geometries found in the Euclidean models cannot be realized in the Lorentzian case.

For the sake of definiteness and simplicity, we will concentrate mostly on the three-dimensional case. The discussion carries over virtually unchanged to $d=4$, the details of which will be given elsewhere [7]. (Obviously, the corresponding continuum theories will be very different, one describing a topological quantum field theory, and the other a field theory of interacting gravitons.) The classical continuum action is simply eq. (1), with $d = 3$. Each discrete Lorentzian space-time will be given by a sequence of two-dimensional compact spatial slices of fixed topology, which for simplicity we take to be that of a two-sphere. Each slice carries an integer "time" label t, so that the space-time topology is $I \times S^2$, with I some real interval. The metric data will be encoded by triangulating this underlying space by three-dimensional simplices with definite edge length assignments. There are two types of edges: "space-like" ones (of length squared $l^2 = a^2 > 0$, with the lattice spacing

$a > 0$) which are entirely contained in a slice $t = const.$, and "time-like" ones (of length squared $l^2 = -\alpha a^2 < 0$) which start at some slice t and end at the next slice $t+1$. This implies that all lattice vertices are located at integer t.

A metric space-time is built up by "filling in" for all times the three-dimensional sandwiches between t and $t+1$. We only consider regular gluings which lead to simplicial *manifolds*. Our basic building blocks are given by three types of Lorentzian tetrahedra,

(1) 3-to-1 tetrahedra (three vertices contained in slice t and one vertex in slice $t+1$): they have three space- and three time-like edges; their number in the sandwich $[t, t+1]$ will be denoted by $N_{31}(t)$;

(2) 1-to-3 tetrahedra: the same as above, but upside-down; the tip of the tetrahedron is at t and its base lies in the slice $t+1$; notation $N_{13}(t)$;

(3) 2-to-2 tetrahedra: one edge (and therefore two vertices) at each t and $t+1$; they have two space- and four time-like edges; notation $N_{22}(t)$.

Each of these triangulated space-times carries a discrete causal structure obtained by giving each time-like link an orientation in the positive t-direction. Then, two lattice vertices are causally related if there is a sequence of positively oriented links connecting the two.

The discretized form of the Lorentzian action (1) is obtained from Regge's prescription for simplicial manifolds [8], see [7] for details. The action is written as a function of the deficit angles around the one-dimensional edges and of the three-dimensional volumes, which in turn can be expressed as functions of the squared edge lengths of the fundamental building blocks. The contribution to the action from a single sandwich $[t, t+1]$ is

$$\Delta S_\alpha(t) = 4\pi ak\sqrt{\alpha} + (N_{31}(t) + N_{13}(t))(akK_1 - a^3\lambda L_1) + N_{22}(t)(akK_2 - a^3\lambda L_2), \quad (2)$$

where

$$\begin{aligned} K_1(\alpha) &= \pi\sqrt{\alpha} - 3\,\mathrm{arcsinh}\,\frac{1}{\sqrt{3}\,\sqrt{4\alpha+1}} - 3\sqrt{\alpha}\,\arccos\frac{2\alpha+1}{4\alpha+1}, \\ K_2(\alpha) &= 2\pi\sqrt{\alpha} + 2\,\mathrm{arcsinh}\,\frac{2\sqrt{2}\,\sqrt{2\alpha+1}}{4\alpha+1} - 4\sqrt{\alpha}\,\arccos\frac{-1}{4\alpha+1}, \\ L_1(\alpha) &= \frac{\sqrt{3\alpha+1}}{12}, \qquad L_2(\alpha) = \frac{\sqrt{2\alpha+1}}{6\sqrt{2}}. \end{aligned} \quad (3)$$

Note that the sandwich action (2) already contains appropriate boundary contributions, such that S is additive under the gluing of contiguous slices. In relation (2), we have used the rescaled cosmological constant, $\lambda = k\Lambda$.

At each time t the physical states $|g\rangle$ are parametrized by piece-wise linear geometries, given by unlabelled triangulations g of S^2 in terms of equilateral Euclidean

triangles. For a finite spatial volume N (counting the triangles in a spatial slice), the number of states is exponentially bounded as a function of N and the vectors $|g\rangle$, defined to be orthogonal, span a finite-dimensional Hilbert space $\mathcal{H}_N$. The transfer matrix $\hat{T}_N$ will act on the Hilbert space

$$H^{(N)} := \bigoplus_{i=N_{\min}}^{N} \mathcal{H}_i, \tag{4}$$

and the states $|g\rangle$ will be normalized according to

$$\langle g_1|g_2\rangle = \frac{1}{C_{g_1}}\delta_{g_1,g_2}, \qquad \sum_g C_g\ |g\rangle\langle g| = \hat{1}. \tag{5}$$

The symmetry factor C_g is the order of the automorphism group of the two-dimensional triangulation g, which for large triangulations is almost always equal to 1. With each step $\Delta t = 1$ we can now associate a transfer matrix $\hat{T}_N$ describing the evolution of the system from t to $t+1$, with matrix elements

$$\langle g_2|\hat{T}_N(\alpha)|g_1\rangle \equiv G_\alpha(g_1, g_2; 1) = \sum_{\text{sandwich}(g_1 \to g_2)} \mathrm{e}^{i\Delta S_\alpha}. \tag{6}$$

The sum is taken over all distinct interpolating three-dimensional triangulations of the "sandwich" with boundary geometries g_1 and g_2, each with a spatial volume $\leq N$. The propagator $G_N(g_1, g_2; t)$ for arbitrary time intervals t is obtained by iterating the transfer matrix t times,

$$G_N(g_1, g_2; t) = \langle g_2|\hat{T}_N^t|g_1\rangle, \tag{7}$$

and satisfies the semigroup property

$$G_N(g_1, g_2; t_1 + t_2) = \sum_g C_g\ G_N(g_2, g; t_2)\ G_N(g, g_1; t_1). \tag{8}$$

The infinite-volume limit is obtained by letting $N \to \infty$ in eqs. (6)-(8).

A brief remark is in order on our notion of "time": the label t is to be thought of as the discretized analogue of proper time, as experienced by an idealized collection of freely falling observers. We do not claim that this is a distinguished notion of time for pure quantum gravity, but it is *a* possible choice, in the present case suggested by our regularization. Note that in *continuum* formulations the proper time gauge is not usually considered, because it is a gauge choice that – considered for arbitrary geometries – goes bad in an arbitrarily short time. This problem does not occur in the discrete case: by construction we only sum over space-time geometries for which there is a globally well-defined (discrete) "proper time".

The action S associated with an entire space-time $S^1 \times S^2$ of length t in time-direction is obtained by summing expression (2) over all $t' = 1, 2, \ldots t$ and identifying the two boundaries. The result can be expressed as a function of three geometric

"bulk" variables, for example, the total number of vertices N_0, the total number of tetrahedra N_3 and t,

$$\begin{aligned} S_\alpha(N_0,N_3,t) &= N_0\left(4ak(K_1-K_2)-4a^3\lambda(L_1-L_2)\right)+N_3\left(akK_2-a^3\lambda L_2\right) \\ &\quad +t\left(4ak(\pi\sqrt{\alpha}-2(K_1-K_2))+8a^3\lambda(L_1-L_2)\right). \end{aligned} \tag{9}$$

Because of the well-known inequality $N_0 \leq (N_3+10)/3$, valid for all closed three-dimensional simplicial manifolds, this implies the boundedness of the discretized Lorentzian action at fixed three-volume. This result is analogous to what happens in Euclidean dynamical triangulations. We write the partition function as

$$Z_\alpha(k,\lambda,t) = \sum_{T\in\mathcal{T}_t(S^1\times S^2)} \mathrm{e}^{iS_\alpha(N_0(T),N_3(T),t(T))}, \tag{10}$$

with $\mathcal{T}_t(S^1\times S^2)$ denoting the set of all Lorentzian triangulations of $S^1\times S^2$ of length t. It will turn out that a necessary condition for the existence of a meaningful continuum limit is the exponential boundedness of the number of possible triangulations as a function of the space-time volume N_3: only if the growth is at most exponential in N_3, can this divergence potentially be counterbalanced by a cosmological constant term exponentially damped in N_3. In our case, exponential boundedness follows trivially from the same property for Euclidean triangulations (where it has been proven rigorously for $d=3,4$ [9, 10]), since the Lorentzian space-times form a subset of the former. Note that the convergence of the partition function implies the absence of divergent "conformal modes".

As it stands, the sum (10) over complex amplitudes has little chance of converging, due to the contributions of an infinite number of triangulations with arbitrarily large volume N_3. In order to make it well-defined, one must perform a Wick rotation, just as in ordinary quantum field theory. Thanks to the presence of a distinguished global time variable in our model, we can associate a unique Euclidean triangulated space-time with every Lorentzian history contributing in (9). It is obtained by taking the *same* topological triangulation and changing the squared lengths of all time-like edges from $-\alpha a^2$ (Lorentzian) to αa^2 (Euclidean), leaving the space-like edges unchanged. We can then use Regge's prescription for calculating the (real) Euclidean action $S^{\mathrm{E}}_\alpha(N_0,N_3,t)$ associated with the resulting Euclidean metric space-time (where α is always taken to be positive). After some algebra one verifies that by a suitable analytic continuation in the complex α-plane from positive to negative real α, the Euclidean and Lorentzian actions are related by

$$S_{-\alpha}(N_0,N_3,t) = iS^{\mathrm{E}}_\alpha(N_0,N_3,t), \tag{11}$$

for $\alpha > \frac{1}{2}$. This latter inequality has its origin in the triangle inequality for Euclidean triangles: S^{E} is real only for $\alpha \geq \frac{1}{2}$, and the limit $\alpha=\frac{1}{2}$ corresponds to the degenerate case of totally collapsed triangles. Moreover, $\alpha=-1$ is the only point on the real axis in which the coefficient of t in the Lorentzian action (9) vanishes (this corresponds

to $\alpha=1$ in eq. (11)). In this case one rederives the familiar expression employed in equilateral Euclidean dynamical triangulations, namely,

$$\frac{1}{i}S_{-1} \equiv S_1^{\mathrm{E}} = -ak(2\pi N_1 - 6N_3 \arccos\frac{1}{3}) + a^3\lambda N_3 \frac{1}{6\sqrt{2}}. \tag{12}$$

Our strategy for evaluating the partition function is now clear: for any choice of $\alpha > \frac{1}{2}$, continue (9) to $-\alpha$, so that

$$\sum_{T\in\mathcal{T}_t(S^1\times S^2)} \mathrm{e}^{iS_\alpha(N_0,N_3,t)} \quad \stackrel{\alpha\to-\alpha}{\longrightarrow} \quad \sum_{T\in\mathcal{T}_t(S^1\times S^2)} \mathrm{e}^{-S_\alpha^{\mathrm{E}}(N_0,N_3,t)}. \tag{13}$$

Because of the exponential boundedness, the Wick-rotated Euclidean state sum in (13) *is* now convergent for suitable choices of the bare couplings k and λ. We can therefore proceed in two ways: either attempt to perform the sum analytically, by solving the combinatorics of possible causal gluings of the tetrahedral building blocks (as has been done in $d=2$ [4]), or use Monte-Carlo methods to simulate the system at finite volume. Once the continuum limit has been performed, we can rotate back to Lorentzian signature by an analytic continuation of the continuum proper time T (which in the case of canonical scaling is of the form $T=at$; not to be confused with the triangulation T) to iT. If we are only interested in vacuum expectation values of time-independent observables and the properties of the Hamiltonian, we do not need to perform the Wick rotation explicitly, just as in usual Euclidean quantum field theory.

Let us now establish some properties of the discrete real transfer matrix $\hat{T} \equiv \hat{T}(\alpha=-1)$ of our model that are necessary for the existence of a well-defined Hamiltonian $\hat{h}$, such that $\hat{T} = \mathrm{e}^{-a\hat{h}}$. These will be useful in any proof of the existence of a self-adjoint *continuum* Hamiltonian $\hat{H}$. It is difficult to imagine boundedness and positivity arising in the limit from regularized models that do not have these properties.

We will show that $\hat{T}_N$ is symmetric, bounded for finite spatial volume N, and strictly positive. Symmetry is obvious by inspection. The sandwich action (2) is symmetric under the exchange of in- and out-states (corresponding to $N_{31} \leftrightarrow N_{13}$). So is (6), because the counting of interpolating states does not depend on which of the geometries g_1, g_2 defines the in-state, say. The boundedness of $\hat{T}_N$ for finite spatial volume N follows from the finite dimensionality of the Hilbert space $\mathcal{H}^{(N)}$ it acts on and the fact that there is only a *finite* number of possibilities to interpolate between two given spatial triangulations g_1 and g_2 in one step. Positivity, $\hat{T}_N \geq 0$, follows from the reflection positivity of our model under reflection with respect to planes of constant t, for both integer and half-integer t [7] (for regular lattices, these are also referred to as site-reflection and link-reflection, c.f. [11]). However, for the existence of a Hamiltonian we must show that zero cannot occur as an eigenvalue. It suffices to show that

$$\langle g_1|\hat{T}_N^2|g_1\rangle \equiv \sum_g C_g \,\langle g_1|\hat{T}_N|g\rangle\langle g|\hat{T}_N|g_1\rangle > 0 \tag{14}$$

for all states $|g_1\rangle$. From this it would follow that $\hat{T}_N^2 > 0$, which together with the positivity $\hat{T}_N \geq 0$ would imply the desired result. Since the right-hand side of (14) is a sum of positive terms, we must show that for each $|g_1\rangle$ there is at least one state $|g\rangle$ with $\langle g|\hat{T}_N|g_1\rangle > 0$. It is straightforward to show that a possible choice is given by $|g\rangle = |g_{\min}\rangle$, where $g_{\min}$ is the minimal triangulation of two-volume 4 of the two-sphere (the surface of a tetrahedron). We deduce that the transfer matrix is strictly positive, $\hat{T}_N > 0$, and that for finite triangulations there is a self-adjoint Hamiltonian operator $\hat{h}_N$ which is bounded from below.

It should be emphasized that although the summation in the path integral is performed in the "Euclidean sector" of the theory, our construction is not *a priori* related to any path integral for Euclidean gravity proper. The point, already made in the two-dimensional case [4], is that we sum only over a selected class of geometries, which are equipped with a causal structure. Such a restriction incorporates the Lorentzian nature of gravity and has no analogue in Euclidean gravity. We therefore expect our Lorentzian statistical mechanics model to have a totally different phase structure from that of Euclidean dynamical triangulations. This expectation is corroborated by an analysis of the "extreme phases" of Lorentzian quantum gravity, to determine which configurations dominate the path integral

$$Z_\alpha^{\rm E}(k,\lambda,t) = \sum_{T\in\mathcal{T}_t(S^1\times S^2)} {\rm e}^{-S_\alpha^{\rm E}}, \tag{15}$$

for either very small or very large inverse Newton's constant $k > 0$. In order to make a direct comparison with the Euclidean analysis [12, 3], we set without loss of generality $\alpha = 1$ in eq. (15) and rewrite the Euclidean action (12) as

$$S_1^{\rm E} = k_3 N_3 - k_1 N_1, \tag{16}$$

with the couplings

$$k_1 = 2\pi a k, \qquad k_3 = 6\ ak \arccos\frac{1}{3} + a^3\lambda\frac{1}{6\sqrt{2}}. \tag{17}$$

In the thermodynamic limit $N_3 \to \infty$, and assuming a scaling behaviour such that $t/N_3 \to 0$, one derives kinematical bounds on the ratio of links and tetrahedra, $\xi := N_1/N_3$, namely,

$$1 \leq \xi \leq \frac{5}{4}. \tag{18}$$

This is to be contrasted with the analogous result in the Euclidean case, where $1 \leq \xi \leq \frac{4}{3}$. It implies that the branched-polymer (or "stacked-sphere") configurations, which are precisely characterized by $\xi = \frac{4}{3}$, and which dominate the Euclidean state sum at large k_1, cannot be realized in the Lorentzian setting. The opposite extreme, at small k_1, is associated with the saturation of the inequality

$$N_1 \leq \frac{1}{2}N_0(N_0-1), \tag{19}$$

and in the Euclidean theory goes by the name of "crumpled phase". At equality, every vertex is connected to every other vertex, corresponding to a manifold with a very large Hausdorff dimension. Again, it is impossible to come anywhere near this phase in the continuum limit of the Lorentzian model. Instead of (19), we have now separate relations for the numbers $N_1^{(\mathrm{sl})}$ and $N_1^{(\mathrm{tl})}$ of space- and time-like edges,

$$N_1^{(\mathrm{sl})} = \sum_t (3N_0(t) - 6) = 3N_0 - 6t, \qquad N_1^{(\mathrm{tl})} \leq \sum_t N_0(t)N_0(t+1). \tag{20}$$

Assuming canonical scaling, the right-hand side of ineq. (19) behaves like $(\mathrm{length})^6$, whereas the second relation in (20) scales only like $(\mathrm{length})^5$.

We conclude that the phase structure of Lorentzian gravity must be very different from that of the Euclidean theory. In particular, the extreme branched-polymer and crumpled configurations are not allowed in the Lorentzian theory. This is another example of causal structure acting as a "regulator" of geometry. It also raises the hope that the mechanism governing the phase transition will be different, and potentially lead to a non-trivial continuum theory, in three as well as in four dimensions.

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References

[1] for recent reviews, see J. Ambjørn, J. Jurkiewicz and R. Loll: Lorentzian and Euclidean Quantum Gravity – Analytical and Numerical Results, *preprint* NBI-HE-00-02, AEI-2000-3, hep-th/0001124; R. Loll: Discrete approaches to quantum gravity in four dimensions, *Living Reviews in Relativity* **13** (1998), http://www.livingreviews.org, gr-qc/9805049.

[2] P. Bialas, Z. Burda, B. Petersson and J. Tabaczek: Appearance of mother universe and singular vertices in random geometries, *Nucl. Phys. B* **495** (1997) 463-476 [hep-lat/9608030]; P. Bialas, Z. Burda and D. Johnston: Condensation in the backgammon model, *Nucl. Phys. B* **493** (1997) 505-516.

[3] J. Ambjorn, M. Carfora, D. Gabrielli and A. Marzuoli: Crumpled triangulations and critical points in 4D simplicial quantum gravity, *Nucl. Phys. B* 542 (1999) 349-394, hep-lat/9806035.

[4] J. Ambjørn and R. Loll: Non-perturbative Lorentzian quantum gravity, causality and topology change, *Nucl. Phys. B* 536 (1998) 407-434, hep-th/9805108.

[5] J. Ambjørn, K.N. Anagnostopoulos and R. Loll: A new perspective on matter coupling in 2d quantum gravity, *Phys. Rev. D* 60 (1999) 104035, hep-th/9904012.

[6] J. Ambjørn, K.N. Anagnostopoulos and R. Loll: Crossing the c=1 barrier in 2d Lorentzian quantum gravity, *Phys. Rev. D*, to appear, hep-lat/9909129.

[7] J. Ambjørn, J. Jurkiewicz and R. Loll, to appear

[8] T. Regge: General relativity without coordinates, *Nuovo Cim. A* 19 (1961) 558-571; R. Sorkin: Time-evolution in Regge calculus, *Phys. Rev. D* 12 (1975) 385-396; Err. *ibid.* 23 (1981) 565.

[9] M. Carfora and A. Marzuoli: Entropy estimates for simplicial quantum gravity, *J. Geom. Phys.* **16** (1995) 99-119; Holonomy and entropy estimates for dynamically triangulated manifolds, *J. Math. Phys.* **36** (1995) 6353-6376.

[10] J. Ambjørn, M. Carfora and A. Marzuoli: *The geometry of dynamical triangulations*, Lecture Notes in Physics, New Series, m50, Springer, Berlin, 1997, hep-th/9612069.

[11] I. Montvay and G. Münster: *Quantum fields on a lattice*, Cambridge University Press, Cambridge, UK, 1994.

[12] D. Gabrielli: Polymeric phase of simplicial quantum gravity, *Phys. Lett. B* 421 (1998) 79-85, hep-lat/9710055.