Summer School on

Quantum Field Theory

— from a Hamiltonian point of view

2–9 August, 2000

Sandbjerg Manor

Denmark

Supported by

- MaPhySto — Centre for Mathematical Physics and Stochastics, University of Aarhus
- The EU Network on Partial Differential Equations and Quantum Mechanics (ERBFM-RXCT960001).

Contents

1 School Program

Most afternoons time is left open for discussion, questions, problem sessions, or simply for studying. On the program this is marked as discussion.

	Wed Aug 2	Thur Aug 3	Fri Aug 4	Sat Aug 5
8:00- 9:00		Breakfast	Breakfast	Breakfast
9:00- 9:50		Spohn	Spohn	Spohn
10:00-10:50		Bach	Bach	Derezinski
11:00-11:50		Bach	Bach	Derezinski
12:00-13:30		Lunch	Lunch	Lunch
13:30-14:00	Arrival	Teufel	Siedentop	Griesemer
14:00-17:00	Arrival	Discussion	Discussion	Time off
17:00-17:30	Arrival	Tadie	Roehrl	Time off
18:00-20:00	Dinner	Dinner	Banquet	Dinner
20:00-21:00	Opening/ Intro of speakers			

	Sun Aug 6	Mon Aug 7	Tues Aug 8	Wed Aug 9
8:00- 9:00	Breakfast	Breakfast	Breakfast	Breakfast
9:00- 9:50	Derezinski	Loss	Hoppe	Graf
10:00-10:50	Graf	Hoppe	Loss	Hoppe
11:00-11:50	Graf	Hoppe	Loss	Discussion
12:00-13:30	Lunch	Lunch	Lunch	Lunch
13:30-14:00	Excursion	Spitzer	Balinsky	Departure
14:00-17:00	from	Discussion	Discussion	
17:00-17:30	13-18	Ammari	Iantchenko	
18:00-20:00	Dinner	Dinner	Dinner	
20:00-21:00			Concluding discussion	

2 Main Lecturers

Volker Bach

Universität Mainz

Spectral Analysis of Nonrelativistic Quantum Electrodynamics

Jan Dereziński

University of Warsaw

Spectral analysis of simple models of Quantum Field Theory

Gian-Michele Graf

ETH Zürich

Stability of (ultraviolet cutoff) non-relativistic QED

Jens Hoppe

Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Potsdam

Membranes and Matrix Models

Michael Loss

Georgia Tech

Self-energy of electrons in nonrelativistic QED

Herbert Spohn

TU München

Dynamics of classical charges and their radiation field

3 Abstracts of Participants' Lectures

Zied Ammari

Ecole Polytechnique

Asymptotic Completeness for the Nelson Model

Alexander Balinsky

Cardiff University

On Zero Modes of Pauli Operators

ABSTRACT: Two results are proved for nul $\mathbb{P}_A$, the dimension of the kernel of the Pauli operator $\mathbb{P}_A = \left\{\sigma\cdot\left(\frac{\mathbf{1}}{\mathbf{i}}\mathbf{abla} + \tilde{\mathbf{A}}\right)\right\}^{\mathbf{2}}$ in $[L^2(\mathbb{R}^3)]^2$:
(i) for $|\vec{B}| \in L^{3/2}(\mathbb{R}^3)$, where $\vec{B} = \mathrm{curl}\vec{A}$ is the magnetic field, nul $\mathbb{P}_{tA} = 0$ except for a finite number of values of t in any compact subset of $(0,\infty)$;
(ii) $\left\{\ \vec{B}:\ \text{nul}\ \mathbb{P}_A = 0,\ \ |\vec{B}| \in L^{3/2}(\mathbb{R}^3)\ \right\}$ contains an open dense subset of $[L^{3/2}(\mathbb{R}^3)]^3$.

Marcel Griesemer

University of Alabama at Birmingham

Ground States in Non-relativistic Quantum Electrodynamics

ABSTRACT: The excited states of a charged particle interacting with the quantized electromagnetic field and an external potential all decay, but such a particle should have a true ground state — one that minimizes the energy and satisfies the Schrödinger equation. We prove quite generally that this state exists for *all values* of the fine-structure constant and ultraviolet cutoff. We also show the same thing for a many-particle system under physically natural conditions.

This is joint work with Elliott Lieb and Michael Loss.

Alexei Iantchenko

Malmö Högskola

Asymptotic Behavior of the One-Particle Density Matrix of Atoms at Distances Z^{-1} from the Nucleus

ABSTRACT: We prove that the suitably rescaled density matrix of ground states of atomic Schrödinger atoms with nuclear charge Z converges on the scale $1/Z$ to the projection of the negative spectral subspace of the Schrödinger operator of the hydrogen atom ($Z = 1$).

Norbert Roehrl

Universität München

On the Infimum of the Energy of the Electron-Positron Field

ABSTRACT: Bach et al derived an exact energy functional for the relativistic electron-positron field with second quantized Coulomb interaction in Hartree-Fock approximation. It resembles the standard QED Hamiltonian as found, e.g., in the textbook of Bjorken and Drell without magnetic vector potential ($\mathcal{A} = 0$) up to different normal ordering.

In this model every particle carries the relativistic kinetic energy $\sqrt{c^2p^2 + m^2c^4}$, which includes the rest energy. Thus creating a particle also increases the total energy by the rest energy. We show that if we substract the rest energy, i.e., creating mass will not increase the total energy, there will no longer be a ground state, and the energy per particle will be negative. We give a good estimate on the fraction of the rest energy that can be substracted while the Fock space vacuum still is the ground state.

Heinz Siedentop

Universität München

Renormalization of the Relativistic Electron-Positron Field

Abstract: We consider the relativistic electron-positron field interacting with itself via the Coulomb potential defined with the physically motivated, positive, density-density quartic interaction. The more usual normal-ordered Hamiltonian differs from the bare Hamiltonian by a quadratic term and, by choosing the normal ordering in a suitable, self-consistent manner, the quadratic term can be seen to be equivalent to a renormalization of the Dirac operator. Formally, this amounts to a Bogolubov-Valatin transformation, but in reality it is non-perturbative, for it leads to an inequivalent, fine-structure dependent representation of the canonical anticommutation relations. This non-perturbative redefinition of the electron/positron states can be interpreted as a mass, wave-function and charge renormalization, among other possibilities, but the main point is that a non-perturbative definition of normal ordering might be a useful starting point for developing a consistent quantum electrodynamics.

This is joint work with E.H. Lieb.

Wolfgang Spitzer

University of Copenhagen

A new proof of the Scott correction

Abstract: We present an elementary proof of the asymptotic behavior of the sum of the negative eigenvalues of Schrödinger operators, $H = -h^2\Delta + V$, with Coulomb singularities. This is, for instance, important in the study of large molecules where V is the Thomas-Fermi potential. In this case our result implies the Scott correction for such systems (cf. Ivrii & Sigal).

This is a joint work with Jan Philip Solovej and supported by the TMR EU network program "PDE and quantum mechanics".

Tadie

University of Copenhagen

Symmetry of the ground states of the Thomas-Fermi Theory for Atoms type equations

ABSTRACT: We show that any solution of the problem $\triangle u = u^\gamma; \quad \gamma \in (1,\ n/(n-2))$ in $\mathbb{R}^n, \quad n > 2$ $u \longrightarrow \infty$ at 0 and $u \searrow 0$ as 'x' $\searrow \infty$, is radially symmetric (i.e. $u(x)$ depends only on $|x|$. For $\gamma = 3/2$ this applies to the Thomas-Fermi theory for Atoms.

Stefan Teufel

TU München

Semiclassics for the massive Nelson model

ABSTRACT: We consider a single, nonrelativistic particle coupled to a quantized massive scalar field under the influence of a slowly varying external potential $V(\varepsilon x)$.

For zero external potential and (sufficiently small) fixed total momentum p the system has a unique ground state $\psi(p)$ with ground state energy $E(p)$ that is separated from the rest of the spectrum by a gap. Wave packets formed out of these fixed momentum ground states are called dressed electron states.

We show that in the limit $\varepsilon \to 0$ the time evolution of a dressed electron state is generated by an effective one-particle Hamiltonian $H_1 = E(-i\nabla_x) + V(\varepsilon x)$ for times of order ε^{-1} with an error of order ε. Thus the dressed electron propagates like a single particle with an effective dispersion $E(p)$.

Finally we show how standard methods of semiclassical analysis can be applied to H_1 in the limit of small ε and how these results can be translated back to the original system.

4 Hand-outs/notes

The following (and possibly more) material will be handed out with the registration package:

- This leaflet.
- Some informational material about MaPhySto: Brochure, MaPhySto News No. 4, list of publications from MaPhySto.
- *Spectral Analysis of Nonrelativistic Quantum Electrodynamics*, notes by Volker Bach.
- *Self-energy of electrons in non-perturbative QED*, paper by Elliott H. Lieb and Michael Loss.
- *Dynamics of Charged Particles and Their Radiation Field*, notes by Herbert Spohn.

5 General information

5.1 Address information

Sandbjerg Gods / Sandbjerg Manor
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DK-6400 Sønderborg
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5.2 Use of computers

At the conference centre 4 PC's are available for email reading/writing etc.

5.3 Meals

All meals are included in the price for accommodation.

Breakfast is (self-)served from 8.00 to 9.00.

Lunch is (self-)served from 12.00 to 13.00. Included is **one** soda/beer per participant. Please pay for additional beverages.

Dinner is (self-)served from 18.00 to 19.00. Included is **one** soda/beer per participant. Please pay for additional beverages.

6 Social events

6.1 Banquet on Friday

The Summer School Banquet will take place on Friday evening, at 18.00. If you for some reason cannot partipate in the Banquet then please notify the organisers.

6.2 Excursion on Sunday

From 13.00 to 17.00.
A walk in the surroundings of the Augustenborg Estate.
Coffee/tea and cake at "Fjordhotellet" in Augustenborg.
A guided tour to the museum of Sønderborg Castle, presenting some highlights of the eventful history of South Jutland through 1000 years.

A bus will take us back from Sønderborg to Sandbjerg in order to have dinner. If you plan to stay behind in Sønderborg in order to have dinner on your own, then please inform the organisers.

7 List of participants (revised)

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