François Bardou

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Jumps and heavy tails in three microscopic systems

This talk will be an introduction to current physics trends which show the importance of jump processes and long tailed distributions in microscopic physical systems.

On the one hand, the observations of individual microscopic quantum or classical objects such as atoms or molecules reveal flagrantly the *jump* nature of many stochastic processes. These jumps were previously masked by ensemble averaging and did not appear explicitly in the formalism. Motivated by the new experimental context, new ways of treating stochastic processes for microscopic systems are emerging, often based explicitly on jumps.

On the other hand, it is being realized that, due to non linear, or even singular, dependences of relevant physical variables on fluctuating parameters generically create *long tails* in the distribution of the physical quantities. These tails generate anomalous scaling behaviours through the inapplicability of the law of large numbers.

The talk will give an overview of three recent physics developments: effects of mutations of RNA molecules [JaB2003], fluctuations in quantum tunneling [DRB2002, RDB2003] and laser cooling of atoms In the last case, a fruitful connection has been established between physics results [BBA2002] and mathematical theories [BBJ2000, BaB2003].

Selected references:

[JaG2003] L. Jaeger and F. Bardou, in preparation.

[DRB2002] V. Da Costa, M. Romeo and F. Bardou, *Statistical properties of currents flowing through tunnel junctions*, J. Magn. Magn. Mater. **258-259**, 90-95 (2002).

[RDB2003] M. Romeo, V. Da Costa and F. Bardou, *Broad distribution effects in sums of log-normal random variables*, Eur. Phys. J. B **32**, 513-525 (2003).

[BBA2002] F. Bardou, J. P. Bouchaud, A. Aspect and C. Cohen-Tannoudji, Lévy Statistics and Laser Cooling, Cambridge University Press, Cambridge (2002).

[BBJ2000] O. E. Barndorff-Nielsen, F. E. Benth and J. L. Jensen, *Markov jump processes with a singularity*, Adv. Appl. Prob. **32**, 779-799 (2000).

[BaB2003] O. E. Barndorff-Nielsen and F. E. Benth, to be published.

Marcel Bökstedt

Department of Mathematical Sciences, University of Aarhus

Entanglement and geometric invariant theory

Let H_1 and H_2 be two finite dimensional vector spaces over the complex numbers. A separable state on the tensor product is a Hermitian operator on $H_1 \otimes H_2$ that can be written as a positive combination of tensor products of Hermitian operators on H_1 respectively H_2.

Even if the dimensions of the vector spaces H_1 and H_2 are as low as 3, it is not easy to describe the geometry of the space of separable states.

I consider the special case where the dimensions of H_1 and H_2 equals 3. The non-compact but reductive group $G = SL_3 \times SL_3$ acts on the Hermitian forms. This action preserves the space of separable hermitian operators.

I use the methods of geometric invariant theory to study this particular case, and to relate the separable states to the stable points of the action of G.

Vladimír Bužek

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Quantum state reconstruction and optimal manipulations with quantum informationd

In my talk I will study how information encoded in quantum states can be reconstructed from incomplete experimental data. In particular:
(1) I will show how the *maximum entropy* (MaxEnt) principle can be efficiently used for an estimation of quantum states (i.e., density operators or Wigner functions) on incomplete observation levels, when just a fraction of system observables are measured (i.e., the mean values of these observables are known from the measurement). In particular, I will analyze in detail how the *MaxEnt* principle can be efficiently applied for a reconstruction of quantum states from incomplete tomographic data. I will present reconstruction of

Wigner functions of a single-mode cavity field based on experimental data from a recent experiment of the group of S.Haroche at the Ecole Normale Superior [P.Bertet et al., *Phys. Rev. Lett.* **89**, 200402 (2002)].
(2) When only a *finite* number of identically prepared systems are measured, then the measured data contain only information about frequencies of appearances of eigenstates of certain observables. I will show that in this case states of quantum systems can be estimated with the help of quantum Bayesian inference. I will analyze the connection between this reconstruction scheme and the reconstruction via the MaxEnt principle in the limit of infinite number of measurements.
(3) I will show that the amount of extractable information from finite ensembles is closely related to the fidelity of manipulations that can be performed on physical systems carrying quantum information. Given the bounds imposed by quantum mechanics (all transformations have to be completely positive maps) optimal quantum machines (e.g. universal NOT gates or quantum cloners) can be 'designed'.

Ignacio Cirac

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Entanglement of Gaussian states and spin systems

The entanglement of formation of two particle Gaussian states can be determined by calculating the variance of two specific operators. This quantitiy can be used to determine which are the multipartite Gaussian state, which is invariant under a particular symmetry group, and for which the entanglement of two particular subsystems is maximized. The same question can be asked for qubit systems. In this presentation I will show how to solve these problems and their relation to the entanglement of certain quantum statistical models.

Giacomo Mauro D'Ariano

Department of Electrical and Computer Engineering, Northwestern University,
Evanston, IL 60208

Universal and programmable measuring devices, and quantum calibration

A universal detector acts on a composite quantum system involving an ancilla, and provides the expectation values of any arbitrary operator by just changing the data processing of the outcomes. A programmable detector instead can be programmed to achieve any desired POVM by changing the state of an ancilla. The resources needed to achieve these new kinds of devices are analyzed and quantified, in term of universal machines such as Bell POVM's and control-unitary transformations. Finally a method for giving a quantum calibration of any measuring device with two tomographers and a faithful state is given, with application to a first absolute quantum calibration of a photo-detector.

Christopher A. Fuchs

Bell Labs, Lucent Technologies, Murray Hill, New Jersey 07974

What is the difference between a quantum observer and a weatherman?

Not much. But where there is a difference, there lies quantum theory's most direct statement about properties of the world by itself (i.e., the world without observers or weathermen). In this talk, I will try to make sense of the last two sentences by writing quantum mechanics in terms that only make reference to probability simplexes, rather than Hilbert spaces.

Richard D. Gill

Utrecht University

Quantifying nonlocality-of-entanglement.

One of the uses of entangled states is to do Bell-type experiments. Such an experiment generates data which is incompatible with a classical (”local realist”) physical description of the phenomenon. Many different experiments have been proposed with this aim, and some have also been performed: Bell, Clauser-Horne-Shimony-Holt, Mermin, Hardy, Greenberger-Horne-Zeilinger, Popescu, Acin-Gisin-Latorre, ...

Different experiments will need different numbers of particles in order to get equally convincing evidence against local realism. I will quantify this idea, and show how it leads to a new measure of entanglement, or perhaps better, of non-locality: it turns out that the most non-local behaviour is not always observed with the most entangled state.

Reference: quant-ph/0307125 (Van Dam, Gill and Grünwald)

Madalin Guta

Eurandom, Eindhoven

Sieved maximum likelihood state estimation

We describe quantum tomography as an inverse statistical problem and show how entropy methods can be used to study the behaviour of kernel and sieved maximum likelihood estimators.

Masahito Hayashi

Quantum Computation and Information Project, ERATO, JST, Tokyo

Quantum central limit theorem and quantum estimation

Holevo derived a lower bound of mean square error of locally unbiased estimator in quantum estimation. This minimum mean squre error equals to the asymptotic bound of proper estimators. In quantum Gaussian case, the bound by Holevo is achievable. On the other hand, as is proven in quantum central limit theorem, any quantum state goes to quantum Gaussian state. Using this result, I construct an estimator achieving Holevo's bound.

Alexander S. Holevo

Steklov Mathematical Institute, 119991 Moscow

Asymptotic estimation theory of the shift parameter of a quantum state

We develop the estimation theory for the shift parameter in a series of independent copies of a pure quantum state. The optimal covariant estimator is found and its asymptotics is studied. It is shown that under the regularity assumption of finiteness of the second moment of the shift generator, the optimal estimator is asymptotically normal and its variance tends to the absolute lower bound given by the generalized uncertainty relation. On the other hand, the variance of estimators based on unentangled measurements is lowerbounded by a Cramer-Rao bound which can be strictly greater than the bound of uncertainty relation, which is a manifestation of the strict superadditivity of the statistical information in observations of independent quantum systems.

Peter E. Jupp

University of St. Andrews

Some Aspects of Quantum Statistical Inference

In classical parametric statistical inference, an important question is 'What parts of the data are informative about the parameters of interest?'. Key concepts here are those of sufficient statistic, ancillary statistic and cut. This talk will outline some analogous concepts for quantum instruments.

Jan-Åke Larsson

Linköping University

Bell inequalities for position measurements

Bell inequalities for position measurements are derived using the bits of the binary expansion of position-measurement results. Violations of these inequalities are obtained from the output state of the nondegenerate optical parametric amplifier.

Elena R. Loubenets

Department of Mathematical Sciences, University of Aarhus

On the quantum probabilistic model

We develop a new mathematical framework for the probabilistic description of an experiment upon a system of any type in terms of initial information representing this system. Based on this framework, we introduce the probabilistic model for the description of non-destructive experiments upon a quantum system and prove that positive bounded linear mappings on the Banach space of trace class operators, arising in the description of generalized quantum measurements, are completely positive.

Hideo Mabuchi

California Institute of Technology, Pasadena

Continuous measurement, quantum metrology, and quantum information

Recent experimental and theoretical work in our group has focused on the use of continuous measurement and real-time feedback to realize and to exploit novel evolutions for optical and atomic systems. In one experiment we used broadband quantum-noise limited measurement and prompt feedback to implement an "adaptive homodyne measurement" that enables single-shot estimation of the phases of weak coherent-state optical pulses, with lower variance than any previously demonstrated technique. In a second, ongoing experiment, we are investigating continuous measurement of the collective spin of a cold atom ensemble. Here, our interests are in spin squeezing, Heisenberg-limited magnetometry, and dynamic generation of large-scale entanglement. In this talk I will review our recent results and discuss the central role of quantum trajectory theory in formulating optimal strategies for metrology and entanglement-generation via continuous quantum measurement.

Keiji Matsumoto

National Institute of Informatics, Quantum Computation Group, Tokyo 101-8430

Asymptotic Quantum Estimation Theory and Berry-Uhlmann curvature

Recently, asymptotic theory of quantum statistics are studied by several authors, and for some cases, optimal efficiency is known. In this talk, pure state models are mainly stdudied, and relation between asymptotic efficiency and Berry phase (Berry-Uhlmann curvature) is discussed. Mixed state models are also studied.

K.R. Parthasarathy

Indian Statistical Institute, New Delhi

Construction of non-stabilizer error-correcting quantum codes

Starting from the well-known theory of classical codes we shall present a brief introduction to the theory of error-correcting quantum codes with emphasis on the Gottesman stabilizer codes. By pooling several Gottesman codes with different character coefficients we shall construct examples of nonstabilizer codes. The connection between the codes of classical information theory and quantum codes on the one hand and the theory of entanglement of states on the other will also be highlighted.

Asher Peres

Department of Physics, Technion - Israel Institute of Technology, Haifa

Quantum Information and Relativity Theory

Quantum mechanics, information theory, and relativity theory are the basic foundations of theoretical physics. The acquisition of information from a quantum system is the interface of classical and quantum physics. Special relativity imposes severe restrictions on the transfer of information between distant systems. Various applications will be presented.

Manuel A. Ballester Sanchez

Utrecht University

Estimation of unitary quantum operation

The problem of optimally estimating an unknown unitary quantum operation is treated. A comparison is made between separable and non-separable measurements and it is shown that non-separable measurements are always better by at least a factor of two. It is also shown that if separable measurements are used, classical communication does not improve

the estimation. In two dimensions it is also shown that measuring one of the Bell states against the other three is as good as any separable measurement and has the advantage of having only two outcomes.

Reference: quant-ph/0305104

Robert Spekkens

Perimeter Institute for Theoretical Physics, Waterloo

In defense of the view that quantum states are states of knowledge

I present a toy theory that is based on a simple information-theoretic principle, namely, that when one has maximal knowledge of a system, one's knowledge is quantitatively equal to one's ignorance. The object analogous to the quantum state in the theory is a uniform probability distribution over hidden states. Because the theory is, by construction, local and non-contextual, it does not reproduce quantum theory. Nonetheless, a wide variety of quantum phenomena have analogues within the toy theory that admit simple and intuitive explanations. Such phenomena include: the non-commutativity of measurements, interference, no information gain without disturbance, the multiplicity of pure state decompositions of a mixed state, the distinction between two-way and intrinsic three-way entanglement, the monogamy of pure entanglement, no cloning, no broadcasting, the remote steering of states, teleportation, dense coding, mutually unbiased bases, locally immeasurable product bases, unextendible product bases, the possibility of key distribution, the impossibility of bit commitment, and many others. The diversity and quality of these analogies provides compelling evidence for the view that quantum states are states of knowledge rather than states of reality, and that maximal knowledge is incomplete knowledge. A consideration of the phenomena that the toy theory fails to reproduce, notably, violations of Bell inequalities and the existence of a Kochen-Specker theorem, provides clues for how to proceed with a research program wherein the quantum state being a state of knowledge is the idea upon which one never compromises.

Jay Gambetta and H. M. Wiseman

Griffith University, Nathan 4111, Australia

The interpretation of non-Markovian stochastic Schrödinger equations: hidden variables

Do diffusive non-Markovian stochastic Schrödinger equations (SSEs) for open quantum systems have a physical interpretation? In a recent paper we investigated this question using the orthodox interpretation of quantum mechanics [Phys. Rev. A **66**, 012108 (2002)]. We found that the solution of a non-Markovian SSE represents the state the system would be in at that time if a measurement was performed on the environment at that time, and yielded a particular result. However, the linking of solutions at different times to make a trajectory is, we concluded, a fiction. In this paper we reinvestigate this question using the modal (hidden variable) interpretation of quantum mechanics. We find that the noise function $z(t)$ appearing in the non-Markovian SSE can be interpreted as the value of a suitably chosen environment property (beable). The solution of the non-Markovian SSE is the state of the system 'conditioned' on this value and the non-Markovian SSE gives its evolution through time. We present the theory for diffusive non-Markovian SSEs that have as their Markovian limit SSEs corresponding to homodyne and heterodyne detection, as well as one which has no Markovian limit.

Christof Wunderlich

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Quantum measurements with individual ions

Individual trapped ions are ideally suited to investigate fundamental issues of quantum mechanics such as the measurement process. Experiments with single Yb^+ ions are reported where an adaptive algorithm for quantum state estimation was implemented, as well as an experiment demonstrating the quantum Zeno paradox with an individual quantum system. In addition, a new type of ion trap is briefly discussed where a collection of ions form an artificial "molecule" with pairwise spin-spin coupling between individual constituents.

The exact determination of the state of a quantum mechanical two-state system (qubit) would require infinitely many measurements on identically prepared qubits. If a finite number N of qubits is available, the optimal estimate of the quantum state is obtained by performing a *simultaneous* measurement on all N qubits. We have performed *sequential* measurements on arbitrary but identically prepared states of a single qubit in order to estimate its state more economically. The base of measurement is varied in real time during a sequence of N measurements, conditioned on the results of previous measurements in this sequence.

The quantum Zeno effect is manifest in another experiment that demonstrates a peculiar prediction of quantum mechanics: Even negative-result measurements (that do not involve a local physical interaction in a classical sense) impede the evolution of a quantum system, here the hyperfine states of Yb^+. The analysis of sequences of such null measurements reveals that the quantum system's evolution is indeed reset to "start" (here state F=0) after each negative result measurement.

In a new type of linear ion trap a magnetic field gradient is applied along the axis of rotational symmetry of the trapping potential. In such a trap the internal states (qubits) of different ions are coupled by an effective spin-spin interaction whose formal description is the same as that for J-coupling in molecules used for nuclear magnetic resonance (NMR) experiments. This coupling can be adjusted by choosing the trap parameters and the type of ion. Thus, an artificial ion "molecule´´ can be designed where individual qubits are well isolated from their environment, can be individually addressed, and read out with nearly unity efficiency.

The $S_{1/2}$ ground-state hyperfine doublet (total angular momentum F=0,1) of a single $^{171}Yb^+$ ion confined in a Paul trap represents the two-state system (qubit) in the experiments reported here. The F=0 $\leftrightarrow$ F=1 transition is driven by a quasiresonant microwave (mw) field at 12.6GHz. Resonance fluorescence on the $S_{1/2}$(F=1) $\leftrightarrow$ $P_{1/2}$ transition driven by a laser at 369 nm serves for state-selective detection with efficiency $\eta > 98\%$.

Jean-Claude Zambrini

University of Lisbon

Euclidean Quantum Mechanics in the momentum representation

We give the basis of a (rigorous) probabilistic interpretation of the momentum representation in quantum mechanics. To each diffusion associated with a quantum state in the position representation corresponds a reversible diffusion with jumps driven by Lévy processes in the momentum representation. The strategy used is strongly inspired by Feynman path integral approach, and should be, in fact, independent of the representation.