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Zero-Range Potentials, Contact Problems, UV Divergencies, and All That

Physics of dilute atomic gases seems to be ideally suited for a description involving zero-range interactions. However, except for the paradigmatic example of the one-dimensional delta-function interacting bosons, any extension of this system to higher dimensions and/or fermionic statistics suffers from a variety of inconsistencies: non-Hermitian interactions, un-normalizable eigenstates, UV divergencies, to name a few. At the same time in the most cases the eigenstate-eigenvalue problem can be replaced by a fully consistent "free" problem with nontrivial boundary conditions, yielding physically relevant exact solutions. The latter, however, does not allow for any kind of a perturbative treatment.

Our current conjecture is that a consistent Hamiltonian formulation of the most zero-range problems is fundamentally impossible, with the practical consequence of unavailability of perturbation theory expansions beyond the first order. We suggest the third, variational formulation of the problem, that retains most of the remaining tools of Quantum Mechanics, such as the first order of the perturbation theory, Hellmann-Feynman theorem, and, possibly, the Born expansion. We treat explicitly the one-dimensional polarized fermions as an example.