The Wiener Sequential Testing Problem with Finite Horizon

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Abstract

We present an explicit solution of the Bayesian problem of sequential testing of two simple hypotheses about the mean value of an observed Wiener process on the time interval with finite horizon. The method of proof is based on reducing the initial optimal stopping problem to a parabolic free-boundary problem where the continuation region is determined by two continuous curved boundaries. By means of the change-of-variable formula containing the local time of a diffusion process on curves we show that the optimal boundaries can be characterized as a unique solution of the coupled system of two nonlinear integral equations.

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