MaPhySto – Centre for Mathematical Physics and Stochastics
Workshop 26 - 28 November, 1998

QUANTUM GEOMETRY, RANDOM MATRICES, STATISTICAL MODELS OF STRINGS AND QUANTUM GRAVITY

Aud. A, The Niels Bohr Institute

Organized by J. Ambjørn, B. Durhuus and J.P. Solovej

As part of the activities of *MaPhysSto* a workshop on quantum geometry, random matrices and statistical models of strings and quantum gravity was organized at the Niels Bohr Institute, University of Copenhagen, November 26-28th, 1998. The workshop was co-sponsored by the Mathematical Institute, University of Copenhagen, via the SNF-grant *Geometry and Global Analysis*, and by the Niels Bohr Institute. The workshop dealt with the above topics in a broad sense, also emphasizing the newest developments in theoretical physics, i.e. M-theory and M(atrix) theory, although they were not explicitly mentioned in the title of the workshop. Below the program and abstracts are listed.

PROGRAM

Thursday, November 5

09:00 - 09:45	**N. Obers** *(Nordita)*: U-duality and M(atrix) theory
09:50 - 10:35	**R. Szabo** *(NBI)*: D-branes and the non-commutative structur of quantum space-time
10:40 - 11:10	Coffee break
11:10 - 11:40	**P. Olesen** *(NBI)*: Heavy quark potential in the AdS/CFT approach
11:45 - 12:15	**L. Vaksman** *(Ukraine)*: q-analogues of some bounded symmetric domains
12:20 - 13:05	**A. Bilal** *(Neuchatal)*: Discrete light-cone quantization of M-theory as a light like limit
13:10 - 14:10	Lunch break
14:10 - 14:40	**C. Thomassen** *(DTU)*: Resistances in electrical networks and random walks on graphs
14:45 - 15:15	**J. Nishimura** *(NBI)*: Dynamical aspects of large N reduced models
15:20 - 16:05	**R. Loll** *(MPI, Potsdam)*: A model of 2d Lorentzian quantum gravity
16:10 - 16:40	Coffee break
16:40 - 17:10	**J.L. Nielsen** *(NBI)*: Hamiltonian cycles on a random Eulerian triangulation
17:15 - 17:45	**J. Wheater** *(Oxford)*: The Hausdorff dimension in polymerized quantum gravity

Friday, November 6

09:00 - 09:45	**U. Haagerup** *(Odense University)*: Asymptotic bounds for spectra of random matrices and random operators on Hilbert space
09:50 - 10:35	**V.A. Kazakov** *(ENS)*: Exactly calculable SUSY matrix integrals
10:40 - 11:10	Coffee break
11:10 - 11:40	**L. Chekhov** *(Steklov)*: String matrix partition functions
11:45 - 12:30	**I. Kovstov** *(Saclay)*: Matrix string partition functions
12:35 - 13:05	**J. Jurkiewicz** *(Krakow)*: Connected correlators in quantum gravity
13:10 - 14:10	Lunch break
14:10 - 14:55	**Yu. Makeenko** *(ITEP)*: Matrix theory at finite temperatur
15:00 - 15:30	**D. Johnston** *(Heriot-Watt)*: Vertex models on random graphs
15:35 - 16:05	**B. Petersson** *(Bielefeld)*: The phase structure of simplicial quantum gravity with a measure factor and the extremal ensemble
16:10 - 16:40	Coffee break
16:40 - 17:10	**G. Thorleifsson** *(Bielefeld)*: The joy of being degenerate
17:15 - 17:45	**K. Anagnostopoulos** *(NBI)*: Can gauge fields change the phase diagram of 4D quantum gravity?

Saturday, November 7

09:00 - 09:45	**F. David** *(Saclay)*: Renormalization group and large orders for self-avoiding surfaces
09:50 - 10:20	**A. Akemann** *(MPI, Heidelberg)*: QCD with and without random matrix models
10:25 - 10:55	Coffee break
11:00 - 11:45	**W. Janke** *(Mainz and Leipzig)*: Spin systems on quenched random graphs and other random lattices
11:50 - 12:35	**M. Carfora** *(SISSA and Pavia)*: The geometry of the strong coupling phase in 4D simplicial gravity
12:40 - 13:40	Lunch break
13:40 - 14:10	**G. Savvidy** *(Demokratos, Athen)*: Discretized loop transfer matrix: Eigenvalues and eigenfunctions
14:15 - 14:45	**T. Jonsson** *(Reyklavik)*: Transfermatrix for loops
14:50 - 15:35	**P. Bialas** *(Bielefeld)*: Crumpling phase transition in random gravity
15:40 - 16:10	Coffee break
16:10 - 16:40	**Z. Burda** *(Krakow and Orsay)*: Phase diagram of the mean field model of simplicial gravity
16:45 - 17:15	**A. D'Adda** *(INFN, Turin)*: YM2 as a matrix string theory

Abstracts

G. Akemann

QCD with and without Random Matrix Models

Using relations from random matrix theory, we derive exact expressions for all n-point spectral correlation functions of Dirac operator eigenvalues in terms of finite-volume partition functions. This is done for both chiral symplectic and chiral unitary random matrix ensembles, which correspond to $SU(N_c \geq 3)$ gauge theories with N_f fermions in the adjoint and fundamental representations, respectively. In the latter case we infer from this an infinite sequence of consistency conditions that must be satisfied by the corresponding finite-volume partition functions.

K. N. Anagnostopoulos

Can $U(1)$ Gauge Fields Change the Phase Diagram of 4d Quantum Gravity?

We consider the effect of $U(1)$ gauge fields coupled to 4d quantum gravity in the context of dynamical triangulations. Numerical simulations are performed to the model where the fields are placed on the dual graph of triangulations. No evidence of significant change of the geometry of space time is found compared to pure gravity even for a large number of gauge fields. Recent results where it was found that a large enough number of gauge fields placed on the links of the triangulation can change the geometry of the weak coupling phase can be understood as being induced by the measure term that one obtains while mapping one model onto the other.

P. Bialas

Crumpling Phase Transition in Random Graphs

In this talk I would to like present two models of random geometries, which exhibit a phase transition between an elongated phase with Hausdorf dimension two and $\gamma = \frac{1}{2}$, and a crumpled phase dominated by an singular vertex and negative γ. The motivation for studying those models is that similar phase transitions happen in higher dimensional simplicial gravity.

First model I want to consider are the Branched Polymers. This model was already well studied and its known that it can exhibit a continues phase transition.

It was shown that putting Ising spins on this ensemble does not affect its properties. I will show that this changes in the presence of the magnetic field. Magnetic field introduces an interaction between the spins and the geometry that is particularly strong in case of the antyferromagnetic coupling. This interaction is strong enough to crumple the trees. The resuting phase transition has different crtitical behaviour the the transition on the bare Branched Polymers.

A will also comment on the correlation functions, and the mean–field approximation of those models.

Z. Burda

Phase Diagram of the Mean Field Model of Simplicial Gravity

We discuss the phase diagram of the mean field model of simplicial gravity with the measure term $p(q) = q^{-\beta}$ on the (κ, β) plane, where q is vertex order and κ is the coupling to the integrated curvature. The system has three phases : *fluid*, *crumpled* and *crinkled*. Both the crinkled and crumpled phases are characterized by the presence of singular vertices. The two phases differ by the average curvature which small in the crumpled and big in the crinkled phase. The transition between them is discontinuous. The order of the transition between the fluid and the crumpled phase depends on β. For $\beta \in (2, \infty)$ it is first order, while for $\beta \in (1, 2]$ it is continuous. We determine the critical exponent for this transition and show that the transition becomes softer when β approaches unity and eventually disappears at $\beta = 1$. The phase diagram of the mean field model is in very good agreement with the results of the Monte Carlo simulations of 4d simplicial gravity with the measure term or vector fields. We generalize the discussion to an arbitrary set of one vertex weights $p(q)$.

A. Bilal

Discrete Ligth-cone Quantization of M-Theory as a Light Like Limit

We investigate whether DLCQ of M-theory can be defined as a limit of M-theory compactified on an almost light-like circle. This is of particular interest since the proofs of the matrix description of M-theory by Seiberg and Sen rely on this assumption. By the standard relation between M-theory on S^1 and IIA string theory, we translate this question into the corresponding one about the existence of the light-like limit of IIA superstring theory for any string coupling g_s. We argue that perturbative string loop amplitudes should have a finite and well-defined light-like limit provided the external momenta are chosen to correspond to a well-defined DLCQ set-up. On the non-perturbative side we consider states and amplitudes. We show that an appropriate class of non-perturbative states (wrapped D-branes) precisely have the right light-like limit. We give some indications that non-perturbative corrections to string amplitudes, too, may behave as required in the light-like limit. Having perturbative and non-perturbative evidence, this suggests that type IIA superstring theory as a whole has a well-defined light-like limit (for any string coupling g_s) and hence that the same is true for M-theory.

M. Carfora

The Geometry of the Strong Coupling Phase in 4D Simplicial Gravity

In this talk, largely based on our work with J. Ambjørn, A. Marzuoli and D. Gabrielli (hep-lat/9806035, to appear in Nuc.Phys.B), we present a detailed geometrical analysis of the strong coupling phase in 4-dimensional simplicial quantum gravity. We characterize explicitly the singular triangulations of the 4-sphere that are closer (with respect to the canonical ensemble measure) to the allowed kinematical boundaries of existence. In particular, we show that the distribution of dynamical triangulations with singular vertices and singular edges, dominating in the strong coupling regime, is characterized by distinct sub-dominating peaks. The presence of such peaks generates volume dependent pseudo-critical points: $k_2^{crit}(N_4 = 32000) \simeq 1.25795$, $k_2^{crit}(N_4 = 48000) \simeq 1.26752$, $k_2^{crit}(N_4 = 64000) \simeq 1.27466$, etc., which appear in good agreement with available Monte Carlo data. It is also possible to estimate analytically the (infinite volume) critical coupling separating the weak and the strong coupling regime in $4D$ simplicial quantum gravity to be located at $k_2^{crit} \simeq 1.3093$. Under a certain scaling hypothesis we analytically characterize the (canonical) average value, $c_1(N_4; k_2) =< N_0 > /N_4$, and the susceptibility, $c_2(N_4; k_2) = (< N_0^2 > - < N_0 >^2)/N_4$, associated with the vertex distribution of the 4-D triangulations considered. Again, the resulting analytical expressions are found in quite a good agreement with their Monte Carlo counterparts.

L. Chekhov

String Matrix Models in 1/N: A Web of Relations

We investigate the newly appeared matrix model with external field related to the string matrix models and find explicit answers in 1/N expansion for the NBI and 2-log matrix models. Comaring Virasoro constraints we prove the equivalence bwteen NBI and Kontsevich as well as between 2-log and the Kontsevich–Penner model (or, equivalently, Hermitian one-matrix model). The solutions in terms of moments can be elaborated to all genera.

A. D'Adda

Two dimensional Yang-Mills theory as a matrix string theory

We quantize pure 2d Yang-Mills theory on a torus in the gauge where the field strength is diagonal. Because of the topological obstructions to a global smooth diagonalization, we find string-like states in the spectrum similar to the ones introduced by various authors in Matrix string theory. These states are necessary if one requires invariance under modular transformations, namely under large diffeomorphisms, and are not present in the conventional formulation of Yang-mills theory on a Riemann surface. We write explicitly the partition function and discuss the large area limit, where it coincides with the partition function of the coverings of the torus (without branch points), versus the small area limit where the large N expansion of Gross and Taylor holds and gives the same description of the partition function in terms of coverings of the torus. The possibility of extending this approach to higher genus Riemann surfaces and to coverings with branch points (interacting strings) is also discussed.

F. David

Renormalization Group and Large Orders for Self-avoiding Surfaces

The large order behavior of the perturbative expansion for the continuous model of tethered self-avoiding membranes is controlled by a classical configuration for an effective potential in bulk space, which is the analog of the Lipatov instanton, solution of a highly non-local equation. The n-th order is shown to have factorial growth as $(-\mathrm{cst})^{\mathrm{n}}\,(\mathrm{n}!)^{(1-\epsilon/\mathrm{D})}$, where D is the "internal" dimension of the membrane and ϵ the engineering dimension of the coupling constant for self-avoidance. The instanton is calculated within a variational approximation, which is shown to become exact in the limit of large dimension d of bulk space. This is the starting point of a systematic $1/d$ expansion. As a consequence, the ϵ-expansion of self-avoiding membranes has a factorial growth, like the ϵ-expansion of polymers and standard critical phenomena, suggesting Borel summability. Consequences for the applicability of the 2-loop calculations are discussed.

W. Janke

Spin Systems on Quenched Gravity Graphs and other Random Lattices

Phase transitions in disordered systems are quite generally expected to exhibit a different behavior than in the corresponding pure system. In this talk I discuss a series of Monte Carlo studies of a special type of such disordered systems, namely quenched, random gravity graphs and Poissonian random lattices of the Voronoi/Delaunay type. In two dimensions we obtain for the 8-state Potts model strong evidence that in the Poissonian case the order of the phase transition does not change, i.e., it is of first order as in the pure system. Using the two-dimensional random lattices appearing in the dynamically triangulated random surfaces (DTRS) approach to quantum gravity, however, we find that the even stronger first-order transition of the 10-state Potts model in the pure system is softened to a continuous phase transition. For the Ising model on three-dimensional Poissonian random lattices our data provide again strong evidence that, for this type of disorder, no qualitative differences to the pure system occur: for the considered system sizes, the critical exponent values agree with recent high-precision Monte Carlo estimates on regular cubic lattices and with field-theoretical calculations. The talk concludes with a comparison of the two random lattice constructions with the case of bond randomness, and a discussion of the distinguishing differences.

D. Johnston

Vertex Models on Random Graphs

We discuss a **4**-vertex model on an ensemble of **3**-valent (Φ^3) planar random graphs, which is equivalent to coupling the vertex model to **2D** quantum gravity. The regular lattice equivalent of the model is the symmetric 8-vertex model on the honeycomb lattice, which displays the critical behaviour of the Ising model in field as was shown by Wu et.al. using weak graph transformation techniques.

We note that the Ising equivalence still holds for the 4-vertex model on Φ^3 graphs, which again allows a determination of the critical behaviour of the vertex model. The relations between the vertex weights and Ising model parameters are unrenormalized by the coupling to gravity, as is the equation of the Ising locus for the vertex weights. The weak graph symmetry of the vertex weights which is fundamental to the solution can be understood in the Φ^3 case as a change of integration variable in the matrix integral used to define the model.

We note the Ising equivalence can still be used to determine the critical behaviour of the spin 1 Blume-Emery-Griffiths model for a particular subset of couplings on Φ^3 graphs, as on the honeycomb lattice, giving a rare exact solution for a higher spin model.

T. Jonsson

Transfermatrix for Loops

We study the transfer matrix for gonihedric random surfaces in a three-dimensional hypercubic lattice in the approximation where the curvature terms in the transfer matrix are ignored. This amounts to dropping the action on surface edges parallel to the time direction. In this case the matrix simplifies drastically and only depends on the symmetric difference of the links in adjacent planes. The two largest eigenvalues of the transfer matrix can be calculated. The largest one equals the partition function of the two-dimensional Ising model and the ratio between the two largest eigenvalues equals the internal energy per link in the two-dimensional Ising model. We develope an approximation scheme to evaluate the higher eigenvalues which is good away from the critical regime of the Ising model.

J. Jurkiewicz

Connected Correlators in Quantum Gravity

We discuss the concept of connected, reparameterization invariant matter correlators in quantum gravity. We analyze the effect of discretization in two solvable cases : branched polymers and two-dimensional simplicial gravity. In both cases the naively defined connected correlators for a fixed volume display an anomalous behavior, which could be interpreted as a long-range order. We suggest that this is in fact only a highly non-trivial finite-size effect and propose an improved definition of the connected correlator, which reduces the effect. Using this definition we illustrate the appearance of a long-range spin order in the Ising model on a two-dimensional random lattice in an external magnetic field H, when $H \to 0$ and $\beta = \beta_C$.

V. Kazakov

Exactly Calculable SUSY Matrix Integrals

We study the regularized correlation functions of the light-like coordinate operators in the reduction to zero dimensions of the matrix model describing D-particles in four dimensions. We investigate in great detail the related matrix model originally proposed and solved in the planar limit by J. Hoppe. It also gives the solution of the problem of 3-coloring of planar graphs. We find interesting strong/weak 't Hooft coupling dependence. The partition function of the grand canonical ensemble turns out to be a tau-function of KP hierarchy. As an illustration of the method we present a new derivation of the large-N and double-scaling limits of the one-matrix model with cubic potential.

I. Kostov

The R-R partition function of Matrix string theory

The Ramond-Ramond partition function of Euclidean ten-dimensional $U(N)$ super-Yang-Mills theory dimensionally reduced to a two-dimensional torus is evaluated quasi-classically. The result is interpreted in terms of free strings wrapping the space-time torus, as expected from the point of view of Matrix string theory. It is demonstrate that, when extrapolated to the ultraviolet limit (small area of the torus), the quasi-classical expressions reproduce exactly the recently obtained expression for the partition function of the completely reduced super-Yang-Mills theory, including the overall numerical factor. This is an evidence that the theory possesses the property of having exact quasiclassics.

R. Loll

A model for 2d Lorentzian quantum gravity

Taking the Lorentzian nature of space-time seriously, one can construct a discrete model of 2d gravity along the lines of the usual dynamical triangulations approach [1,2]. However, each 2-geometry contributing to the path integral is required to satisfy a causality condition, excluding e.g. the local outgrowth of "baby universes". This is motivated by the fact that on such "trouser topologies" in the continuum any Lorentzian metric must possess singular points, and that field propagation on Lorentzian backgrounds near such points is known to become singular.

Also in 2d gravity putting in causality at the discretized level has a drastic effect: a non-trivial continuum limit exists, but the resulting continuum theory does not lie in the same universality class as the usual Euclidean Liouville quantum gravity. The typical quantum geometry is much smoother than in the Euclidean case, in particular, its Hausdorff dimension is 2 rather than 4. Relaxing the causality condition and allowing for a restricted form of baby-universe branching, one rederives (in the Euclidean sector with a real cosmological constant) the results of Liouville gravity.

[1] J. Ambjorn and R. Loll, Nucl. Phys. B 536 (1998) 407-434 [2] J. Ambjorn, R. Loll, J. Nielsen and J. Rolf, hep-th/9806241

Yu. Makeenko

Matrix Theory at Finite Temperature

The Matrix Theory is the large-N limit of the 10-dimensional supersymmetric Yang–Mills theory dimensionally reduced to 0 spatial dimensions. When the coupling constant g is large, the Matrix Theory describes 11-dimensional M-theory while the limit of small g is associated with 10-dimensional IIA superstring. The Matrix Theory correctly reproduces properties of D-branes in the superstring theory including their interactions to the leading order in violation of supersymmetry, e.g. at small velocities or large separations between D-branes or weak magnetic fields living on D-branes.

We consider the formulation of the Matrix Theory at finite temperature given by an Euclidean path integral with boundary conditions along the compactified "time" which are periodic for the Yang–Mills fields and antiperiodic for fermionic superpartners. We compute the effective potential between static D0-branes in the one-loop approximation and show that the result agrees with an analogous computation in superstring theory, where an integration is to be performed over the non-trivial holonomies of the temporal components of Abelian gauge fields living on the D0-branes. This agreement is to the leading order in the supersymmetry violation by temperature, where the one-loop approximation is reliable, thus providing one more argument supporting the validity of the Matrix Theory. The computed effective static potential which is short-ranged and attractive has consequences for thermal properties of D0-branes.

The talk is based on the paper J. Ambjørn, Y. Makeenko and G. Semenoff, *Thermodynamics of D0-Branes in Matrix Theory*, hep-th/9810170.

J. L. Nielsen

Hamiltonian Cycles on a Random Eulerian Triangulation

A random Eulerian triangulation is a random triangulation where an *even* number of triangles meet at any given vertex. We argue that the central charge increases by one if the fully packed $O(n)$ model is defined on a random Eulerian triangulation in stead of an ordinary random triangulation. Considering the case $n \to 0$, this implies that the system of random Eulerian triangulations equipped with Hamiltonian cycles describes a $c = -1$ matter field coupled to 2D quantum gravity as opposed to the system of usual random triangulations equipped with Hamiltonian cycles which has $c = -2$. Hence, in this case one should see a change in the entropy exponent from the value $\gamma = -1$ to the *irrational* value $\gamma = \frac{-1-\sqrt{13}}{6}$ when going from a usual random triangulation to an Eulerian one. A direct enumeration of configurations confirms this change in γ.

J. Nishimura

Dynamical Aspects of Large N Reduced Models

We study the large N reduced model of D-dimensional Yang-Mills theory with special attention to dynamical aspects related to the eigenvalues of the $N \times N$ matrices, which correspond to the space-time coordinates in the IIB matrix model. We first put an upper bound on the extent of space time by perturbative arguments. We perform a Monte Carlo simulation and show that the upper bound is actually saturated. The relation of our result to the SSB of the $U(1)^D$ symmetry in the Eguchi-Kawai model is clarified. We define a quantity which represents the uncertainty of the space-time coordinates and show that it is of the same order as the extent of space time, which means that a classical space-time picture is maximally broken. We develop a $1/D$ expansion, which enables us to calculate correlation functions of the model analytically. The absence of an SSB of the Lorentz invariance is shown by the Monte Carlo simulation as well as by the $1/D$ expansion.

N. Obers

U-Duality and M(atrix) Theory

This talk is intended as a pedagogical introduction to M-theory and to its maximally supersymmetric toroidal compactifications, in the frameworks of 11D supergravity, type II string theory and M(atrix) theory. U-duality is used as the main tool and guideline in uncovering the spectrum of BPS states. We review the 11D supergravity algebra and elementary 1/2-BPS solutions, discuss T-duality in the perturbative and non-perturbative sectors from an algebraic point of view, and apply the same tools to the analysis of U-duality at the level of the effective action and the BPS spectrum, with a particular emphasis on Weyl and Borel generators. We derive the U-duality multiplets of BPS particles and strings, U-duality invariant mass formulae for 1/2- and 1/4-BPS states for general toroidal compactifications on skew tori with gauge backgrounds, and U-duality multiplets of constraints for states to preserve a given fraction of supersymmetry. A number of mysterious states are encountered in D¡=3, whose existence is implied by T-duality and 11D Lorentz invariance. We then move to the M(atrix) theory point of view, give an introduction to Discrete Light Cone Quantization (DLCQ) in general and DLCQ of M-theory in particular. We discuss the realization of U-duality as electric–magnetic dualities of the Matrix gauge theory, display the Matrix gauge theory BPS spectrum in detail, and discuss the conjectured extended U-duality group in this scheme.

P. Olesen

Heavy Quark Potential in the AdS/CFT Approach

We consider contributions to the heavy quark potential, in the AdS/CFT approach to SU(N) gauge theory, which may arise from fluctuations of the associated worldsheet in anti-deSitter space. We find that the contributions have a rich structure, involving $1/L$ terms as well as extrinsic curvature terms.

B. Petersson

The Phase Structure of Simplicial Quantum Gravity and the Maximal Ensemble

Simplicial quantum gravity in three and four dimensions has two phases, separated by a first order transition. For weak (bare Newtons) coupling there exists a branched polymer phase and for strong coupling a crumpled phase with singular vertices. We have shown that by chosing a modified measure one finds another more interesting crinkled phase in the weak coupling region. It is of interest to investigate this phase in more detail, as well as to determine the order of the transitions to the other phases. In the weak-coupling limit, $\kappa \to \infty$, the partition function of simplicial quantum gravity is dominated by an ensemble of triangulations with the ratio N_0/N_D close to the upper kinematical limit. For a combinatorial triangulation of the D–sphere this limit is $1/D$. It is natural to expect that this ensemble will be reflected by the properties of the *maximal* ensemble defined as the ensemble of triangulations *exactly* on the upper kinematical limit *ie* triangulations with the maximal possible value of N_0 for the given volume N_D. The advantage is then that the maximal ensemble is easier to investigate both numerically and analytically. We have investigated the maximal ensemble in three dimensions using both Monte Carlo simulations and a strong-coupling expansion of the partition function. We show a sub-class of maximal ensemble formed by so called D–dimensional *stacked spheres* exhibits remarkably small finite-volume corrections and in special case the partition function of this ensemble can be calculated exactly. We have extended our analyzis to include three-dimensional simplicial gravity with a modified measure. As previously observed in four-dimensional simplicial gravity in the large–κ limit, suitably modified the model exhibits a transition to a *crinkled* phase. We show that the transition is of higher order, and investigate further the nature of this phase. We find negative values of the sting susceptibility exponent and a Hausdorff dimension, which at the transition is near to three.

G. Savvidy

Loop Transfer Matrix and Gonihedric Loop Diffusion

In the articles [3] the authors formulated a model of random surfaces with an action which is proportional to the linear size of the surface. The model has a number of properties which make it very close to the Feynman path integral for a point-like relativistic particle. This is because in the limit when the surface degenerates into a single world line, the action becomes proportional to the length of the path [1].

In addition to the formulation of the theory in the continuum space the system allows an equivalent representation on Euclidean lattices where a surface is associated with a collection of plaquettes [6]. In these lattice spin systems the interface energy coinsides with the linear-gonihedric action for random surfaces. This gives an opportunity for analytical investigations [7, 8] and numerical simulations [10, 9] of the corresponding statistical systems.

Additional understanding of the physical behaviour of the system comes from the analysis of the transfer matrix [7] which describes the propagation of the closed loops-strings P in time direction with an amplitude which is proportional to the sum of the length $l(P)$ of the string and of the total curvature $k(P)$. The functional $k(P)$ is the total curvature of the polygon-loop P which is equal to the number of corners of the polygon (the vertices with self-intersection are not counted) and $l(P)$ is the length of P which is equal to the number of its links.

This transfer martix can be viewed as describing the propagation of the polygon-loop P_1 at time τ to another polygon-loop P_2 at the time $\tau + 1$ [2].

[1] The problems of spiky instability and the convergence of the partition function have been studied in [3, 4, 5].

[2] Layer-to-layer transfer matrices for three-dimensional statistical systems, whose elements are the product of all Boltzmann weight functions of cubes between two adjacent layers have been considered in the literature [2, 11, 12]. Using Yang-Baxter and Tetrahedron equations one can compute the spectrum of the transfer matrix in a number of

The eigenvalues of the transfer matrix $K_\beta(P_1, P_2)$ define all statistical properties of the system and can be found as a solution of the integral equation in the loop space. We have found two *exact* eigenfunctions $\Psi_Q^{(0)}$, $\Psi_Q^{(1)}$ and the corresponding eigenvalues of the transfer matrix [14]. The largest eigenvalue Λ_0 is equal to the partition function of the 2D-Ising ferromagnet [14]. ¿From this result we can deduce that the critical temperature of the three-dimensional system is equal to the one for the 2D-Ising ferromagnet $2\beta_c = ln(\sqrt{2}-1)$, that the specific heat exponent $\alpha = 0$ and from the hyperscaling law $\nu d = 2 - \alpha$ that $\nu = 2/3$. The ratio Λ_1/Λ_0 is the internal energy of the 2D-Ising system.

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R. Szabo

D-branes and the Non-commutative Structure of Quantum Spacetime

A worldsheet approach to the study of non-abelian D-particle dynamics is presented based on viewing matrix-valued D-brane coordinate fields as coupling constants of a deformed sigma-model which defines a logarithmic conformal field theory. The short-distance structure of spacetime is shown to be naturally captured by the Zamolodchikov metric on the corresponding moduli space which encodes the geometry of the string interactions between D-particles. Spacetime quantization is induced directly by the string genus expansion and leads to new forms of uncertainty relations which imply that general relativity at very short-distance scales is intrinsically described by a non-commutative geometry. The indeterminancies exhibit decoherence effects suggesting the natural incorporation of quantum gravity by short-distance D-particle probes.

C. Thomassen

Resistances in electrical networks and random walks on graphs

We consider a graph as an electrical network in which each edge is a resistor of one Ohm, and we consider the question: What is the effective resistance (also called the driving point resistance) between two neighbors in the 2-dimensional grid? The following old argument gives the answer 1/2: First send one Ampere from a vertex x to infinity. Then send one Ampere from infinity to a neighbor y of x. Finally take the superposition of the two currents. A symmetry consideration shows that the current in the edge xy is 1/2, and the above answer follows. The argument is problematic in that it requires an infinite amount of energy to send a current from x to infinity. We discuss the following questions: For which symmectric graphs does the answer make sense? For which graphs does it give the right answer? Ironically, the answer is correct in all cases where the argument makes no sense and incorrect in many cases where the argument does make sense. The only cases where we do not know the answer are the graphs which satisfy a strong isoperimetric inequality and which have only one end, that is, the deletion of any finite vertex set leaves only one infinite component.

G. Thorleifsson

The Joy of being Degenerate

I extend the definition of simplicial quantum gravity in three and four dimensions to include degenerate triangulations, i.e. triangulations allowing distinct simplexes defined by the same set of vertexes, in addition to combinatorial triangulations traditionally used. This is motivated by the observation in two dimensional simplicial gravity that while including degenerate triangulations does not change the universal critical behavior of the model, the finite-size effects are substantially reduced. I demonstrate numerically that the same is true in higher dimensions. In particular, I provide a strong numerical evidence for an exponential bound on the entropic growth of the ensemble of degenerate triangulations in four dimensions, and demonstrate that a discontinuous crumpling transition is observed on degenerate $4D$ triangulations of volume $N_4 \approx 4000$, compared to $N_4 \approx 32000$ needed to observe the corresponding signal using combinatorial triangulations.

J. Wheater

The Hausdorff Dimension in Polymerized Quantum Gravity

We calculate the Hausdorff dimension, d_H, and the correlation function exponent, η, for polymerized two dimensional quantum gravity models. All the exponents satisfy Fisher scaling. If the non-polymerized model has correlation function exponent $\eta_0 > 3$ then at the polymerized critical point $d_H = \gamma^{-1}$ where γ is the susceptibility exponent and $\eta = 1$. This is the same result as for certain non-generic branched polymer models. and suggests that these models may be in the same universality class.