ASYMPTOTIC FAITHFULNESS OF THE QUANTUM $SU(n)$ REPRESENTATIONS OF THE MAPPING CLASS GROUPS

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Abstract. We prove that the sequence of projective quantum $SU(n)$ representations of the mapping class group obtained from the projective flat $SU(n)$-Verlinde bundles over Teichmüller space is asymptotically faithful, that is the intersection over all levels of the kernels of these representations is trivial, whenever the genus of the underlying surface is at least 3. For the genus 2 case, we prove that this intersection is exactly the order two subgroup, generated by the hyper-elliptic involution, in the case of $n = 2$ and for $n > 2$ the intersection is trivial.

1. Introduction

In this paper we shall study the finite dimensional projective *quantum $SU(n)$ representations* of the mapping class group of a genus g surface. These form the only rigorously constructed part of the gauge-theoretic approach to Topological Quantum Field Theories in dimension 3, which Witten proposed in his seminal paper [W1]. We discovered the *asymptotic faithfulness* property for these representations by studying this approach, which we will now briefly describe, leaving further details to section 2, 3 and the references given there.

By applying geometric quantization to the moduli space M of flat $SU(n)$-connections on an oriented closed surface Σ, one gets a certain finite rank bundle over Teichmüller space $\mathcal{T}$, which we will call the *Verlinde* bundle $\mathcal{V}_k$ at level k, where k is any positive integer. The fiber of this bundle over a point $\sigma \in \mathcal{T}$ is $\mathcal{V}_{k,\sigma} = H^0(M_\sigma, \mathcal{L}^k)$, where M_σ is M equipped with a complex structure induced from σ and $\mathcal{L}$ is an ample generator of the Picard group of M_σ.

The main result pertaining to this bundle $\mathcal{V}_k$ is that its projectivization $\mathbb{P}(\mathcal{V}_k)$ supports a natural flat connection. This is a result proved independently by Axelrod, Della Pietra and Witten [ADW] and by

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Hitchin [H]. Now, since there is an action of the mapping class group , of Σ on $\mathcal{V}_k$ covering its action on $\mathcal{T}$, which preserves the flat connection in $\mathbb{P}(\mathcal{V}_k)$, we get for each k a finite dimensional projective representation, say ρ_k, of , , namely on the covariant constant sections of $\mathbb{P}(\mathcal{V}_k)$ over $\mathcal{T}$. This sequence of representations ρ_k is the *quantum* $SU(n)$ *representations* of the mapping class groups.

In this paper we prove *asymptotic faithfulness* of the quantum $SU(n)$ representations ρ_k:

Theorem 1. *We have that*

$$\bigcap_{k=1}^{\infty} \ker(\rho_k) = \begin{cases} \{1\} & g > 2 \text{ or } (g = 2,\ n > 2) \\ \{1, H\} & g = 2,\ n = 2. \end{cases}$$

where g is the genus of the surface Σ and H is the hyperelliptic involution.

This theorem states that for any element ϕ of the mapping class group , , which is not the identity element, there is an integer k such that $\rho_k(\phi)$ is not a multiple of the identity.

The key ingredient in our proof is the use of the endomorphism bundle $\mathrm{End}(\mathcal{V}_k)$ and the construction of sections of this bundle via Toeplitz operators associated to smooth functions on the moduli space M. By showing that these sections are *asymptotically* flat sections of $\mathrm{End}(V_k)$ (see theorem 7 for the precise statement), we prove that any element in the above intersection of kernels acts trivial on the smooth functions on M, hence act by the identity on M (see the proof of Theorem 8). Theorem 1 now follows directly from knowing which elements of the mapping class group acts trivially on the moduli space M.

The abelian case, i.e. the case where $SU(n)$ is replaced by $U(1)$, was considered in [A2], before we consider the case discussed in this paper. In this case, with the use of Theta-functions, explicit expressions for the Toeplitz operators associated to holonomy functions can be obtained. From these expressions it follows that the Toeplitz operators are not covariant constant either, even in this much simpler case (although the relevant connection is the one induced from the L_2-projection as shown by Ramadas in [R1]). However, they are asymptotically covariant, in fact we find explicit perturbations to all orders in k, which in this case, we argue, can be summed and use to create actual covariant constant sections of the endomorphism bundle. The result as far as the mapping class group goes, is that the intersection of the kernels over all k in this case is the Torelli group.

We remark that it is an interesting problem to understand how the Toeplitz operator constructions used in this paper are related to the deformation quantization of the moduli spaces described in [AMR1] and [AMR2]. In the abelian case, the resulting deformation quantization was explicitly described in [A2].

From the above theorem 1 follows directly that,

Corollary 1. *If the quantum $SU(n)$ representations ρ_k are unitary, then we have for all $\phi \in$, that*

$$|\operatorname{Tr}(\rho_k(\phi))| \leq \dim \rho_k$$

with equality for all k, if and only if

$$\phi \in \begin{cases} \{1\} & g > 2 \text{ or } (g = 2,\ n > 2) \\ \{1, H\} & g = 2,\ n = 2. \end{cases}$$

By the work of Laszlo [La1], it is known that $\mathbb{P}(\mathcal{V}_k)$ with its flat connection is isomorphic to the bundle of conformal blocks for $sl(n, C)$ with its flat connection over $\mathcal{T}$ as constructed by Tsuchiya, Ueno and Yamada [TUY]. This means that the quantum $SU(n)$ representations ρ_k is the same sequence of representations as the one arising from the space of conformal blocks for $sl(n, C)$.

By the work of Reshetikhin-Turaev, Topological Quantum Field Theories have been constructed in dimension 3 from the quantum group $U_q sl(n, C)$ (see [RT1], [RT2] and [T]) or alternatively from the Kauffman bracket and the Homfly-polynomial by Blanchet, Habegger, Masbaum and Vogel (see [BHMV1], [BHMV2] and [B1]).

In ongoing work of Ueno joint with this author (see [AU1] and [AU2]), we are in the process of establishing, that the TUY construction of the bundle of conformal blocks over Teichmüller space for $sl(n, C)$ gives a modular functor, which in turn gives a TQFT, which is isomorphic to the $U_q sl(n, C)$-Reshetikhin-Turaev TQFT. A corollary of this is that the quantum $SU(n)$ representations are isomorphic to the ones that are part of the $U_q sl(n, C)$-Reshetikhin-Turaev TQFT. Since it is well known that the Reshtikhin-Turaev TQFT is unitary one will get unitarity of the quantum $SU(n)$ representations from this. We note that unitarity is not clear from the geometric construction of the quantum $SU(n)$ represention s. Furthermore, one will get that the norm of the Reshtikhin-Turaev quantum invariant for all k and $n = 3$ can separate the mapping torus of the identity from the rest of the mapping tori as a purely TQFT consequence of Corollary 1.

This paper is organized as follows. In section 2 we give the basic setup of applying geometric quantization to construct the Verlinde bundle over Teichmüller space. In section 3 we review the construction of the flat connection in the projective Verlinde bundle. In section 4 we review the general results about Toeplitz operators, we need. In section 5 we prove that the Toeplitz operators for smooth functions on the moduli space give asymptotically flat sections of the endomorphism bundle of the Verlinde bundle. Finally, in section 6 we prove the asymptotic faithfulness Theorem 1 above.

After the completion of this work, we have learned from private communication with Freedman and Walker, that they have a proof of the Asymptotic faithfulness property for the SU(2)-BHMV-representations which uses BHMV-technology.

For the SU(2)-BHMV-representations it is already known by the work of Roberts [Ro], that they are irreducible for k large enough prime and that they have infinite image for k large enough by the work of Masbaum [M].

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2. The gauge theory construction

Let Σ be a closed oriented surface of genus $g \geq 2$ and $p \in \Sigma$. Let P be a principal $\mathfrak{G} = SU(n)$-bundle over Σ. Clearly, all such P are trivializable. Let ξ be an element of the center of $\mathfrak{G}$. Let M be the moduli space of flat $\mathfrak{G}$-connections in $P|_{\Sigma - p}$ with holonomy ξ around p. We can identify

$$M = \mathrm{Hom}_\xi(\pi_1(\Sigma - p), \mathfrak{G})/\, \mathfrak{G},$$

where Hom_ξ means the space of homomorphism of $\pi_1(\Sigma - p)$ to $\mathfrak{G}$ which sends a little loop around p to ξ.

We choose an invariant bilinear form $\{\cdot,\cdot\}$ on $\mathfrak{g} = \mathrm{Lie}(\mathfrak{G})$, normalized such that $-\frac{1}{6}\{\vartheta \wedge [\vartheta \wedge \vartheta]\}$ is a generator of the image of the integer cohomology in the real cohomology in degree 3 of $\mathfrak{G}$, where ϑ is $\mathfrak{g}$-valued Mauer-Cartan 1-form on $\mathfrak{G}$.

This bilinear form induces the structure of a symplectic stratified space on M. Let $M' \subset M$ consist of the irreducible representations. It is well-known that M' is smooth and dense in M and represents the top stratum in this stratification. In fact

$$T_{[A]}M' \cong H^1(\Sigma, d_A),$$

where A is any flat connection in P representing a point in M' and d_A is the induced covariant derivative in the associated adjoint bundle. Using this identification, the symplectic structure on M' is very easily described:

$$\omega(\varphi_1, \varphi_2) = \int_\Sigma \{\varphi_1 \wedge \varphi_2\},$$

where φ_i are d_A-closed 1-forms on Σ with values in $\operatorname{ad} P$.

We have a natural action of , , the mapping class group of Σ, on M induced by pull back. Following Ramadas, Singer and Weitsman [RSW] and Freed [Fr], we will now construct a Hermitian line bundle with a connection $\mathcal{L}$ over M with a natural lift of the , -action on M. We only review the case $\xi = 1$ here and refer the reader to [Fr] for the general case.

Choose a trivialization of P. Let $\mathcal{A}$ be the space of $SU(n)$-connections in P and $\mathcal{A}_F$ the subset of flat connections. The trivialization of P gives a base point A_0 in $\mathcal{A}$. Let $\tilde{\mathcal{L}} = \mathcal{A} \times \mathbb{C}$ with the natural Hermitian structure inherited from $\mathbb{C}$. Let θ be the 1-form on $\mathcal{A}$ given by

$$\theta_A(a) = \int_\Sigma \{(A - A_0) \wedge a\},$$

where $a \in T_A\,\mathcal{A} = \Omega^1(\Sigma, \operatorname{ad} P)$. Then θ determines a Hermitian connection in $\tilde{\mathcal{L}}$. Let

$$\Theta : \mathcal{A} \times \mathcal{G} \to U(1)$$

be the Wess-Zumino-Witten cocycle, see [Fr]. We remark in passing that the WZW-cocycle can be expressed in terms of the Chern-Simons functional. Now, define

$$\mathcal{L} = (\tilde{\mathcal{L}}\,|_{\mathcal{A}_F}) / \,\mathcal{G}$$

where $\mathcal{G}$ acts on $\tilde{\mathcal{L}}$ by

$$g(A, z) = (g^* A, \Theta(A, g) z).$$

The Hermitian connection is invariant under this lift of the action of $\mathcal{G}$ on $\mathcal{A}$, hence we get a topological line bundle over M, which is smooth over M' and a Hermitian connection ∇ in $\mathcal{L}\,|_{M'}$.

By an almost identical construction, we can lift the action of , on M to act on $\mathcal{L}$ such that the Hermitian connection is preserved (See e.g. [A1]).

Let now $\sigma \in \mathcal{T}$ be a complex structure on Σ. Let us review how σ induces a complex structure on M which is compatible with the symplectic structure on this moduli space. The complex structure on Σ induces a $*$-operator on 1-forms and via Hodge theory we get that

$$H^1(\Sigma, d_A) \cong \ker(d_A + * d_A *).$$

On this kernel, consisting of the harmonic 1-forms with values in $\operatorname{ad} P$, the $*$-operator acts and its square is -1, hence we get an almost complex structure on M' by letting $I = -*$. It is a classical result by Seshadri, that this actually makes M' a smooth Kähler manifold, which as such, we denote M'_σ. By using the $(0,1)$ part of ∇ in $\mathcal{L}$, we get an induced holomorphic structure in the bundle $\mathcal{L}|_{M'}$.

From a more algebraic geometric point of view, we let M_σ be the moduli space of S-equivalence class of semi-stable bundles of rank n and determinant isomorphic to $d[p]$, where d is an integer representing ξ under the natural identification of the center of $SU(n)$ with $\mathbb{Z}/n\mathbb{Z}$. By using Mumford's Geometric Invariant Theory, Narasimhan and Seshadri (see [NS]) showed that M_σ is a complex algebraic projective variety homeomorphic to M, such that the quasi-projective sub-variety M_σ^s consisting of stable bundles is isomorphic as a Kähler manifold to M'_σ. Referring to [DN] we recall that

Theorem 2 (Drezet & Narasimhan)**.** *The Picard groups of M_σ and M'_σ agree, i.e.*

$$\operatorname{Pic}(M_\sigma) = \operatorname{Pic}(M'_\sigma) = \langle \mathcal{L} \rangle,$$

where $\mathcal{L}$ is the holomorphic line bundle over M'_σ constructed above.

It follows from this theorem, that the line bundle $\mathcal{L}$ extends to all of M_σ and as such we also denote it $\mathcal{L}$. By Hartog's theorem it follows that

$$H^0(M'_\sigma, \mathcal{L}^k) \cong H^0(M_\sigma, \mathcal{L}^k),$$

for all integers k. In particular, we see that this is a finite dimensional vector space.

Definition 1. *The* Verlinde bundle *$\mathcal{V}_k$ over Teichmüller space, is by definition, the bundle, whose fiber over $\sigma \in \mathcal{T}$ is $H^0(M_\sigma, \mathcal{L}^k)$, where k is a positive integer.*

3. The projectively flat connection

In this section we will review Axelrod, Della Pietra and Witten's and Hitchin's construction of the projective flat connection over Teichmüller space in the Verlinde bundle.

Let $\mathcal{H}_k$ be the trivial $C^\infty(M, \mathcal{L}^k)$-bundle over $\mathcal{T}$ which contains $\mathcal{V}_k$, the Verlinde sub-bundle. If we have a smooth one-parameter family of complex structures σ_t on Σ, then that induces a smooth one-parameter family of complex structures on M, say I_t. In particular we get $\sigma'_t \in T_{\sigma_t}(\mathcal{T})$, which gives an $I'_t \in H^1(M, T)$ (here T refers to the holomorphic tangent bundle of M).

Suppose s_t is a corresponding smooth one-parameter family in $C^\infty(M, \mathcal{L}^k)$ such that $s_t \in H^0(M_{\sigma_t}, \mathcal{L}^k)$. By differentiating the equation

$$(1 + iI_t)\nabla s_t = 0,$$

we see that

$$iI_t'\nabla s + (1 + iI_t)\nabla s_t' = 0.$$

Hence, if we have an operator

$$u(v) : C^\infty(M, \mathcal{L}^k) \to C^\infty(M, \mathcal{L}^k)$$

for all $v \in T(\mathcal{T})$, varying smoothly with respect to v, and satisfying

$$iI_t'\nabla^{1,0}s_t + \nabla^{0,1}u(\sigma_t')(s_t) = 0,$$

then we get a connection induced in $\mathcal{V}_k$ by letting

$$\hat{\nabla}_v = \hat{\nabla}^t_v - u(v), \tag{1}$$

for all $v \in T(\mathcal{T})$, where $\hat{\nabla}^t$ is the trivial connection in $\mathcal{H}_k$.

In order to specify the particular u we are interested in, we use the symplectic structure on $\omega \in \Omega^{1,1}(M)$ to define the tensor $G = G(v) \in \Omega^0(M, S^2(T))$ by

$$I' = G\omega.$$

Following Hitchin, we give an explicit formula for G in terms of $v \in T(\mathcal{T})$:

The holomorphic tangent space to Teichmüller space at $\sigma \in \mathcal{T}$ is given by

$$T^{1,0}_\sigma(\mathcal{T}) \cong H^1(\Sigma_\sigma, K^{-1}).$$

Furthermore, the holomorphic co-tangent space to the moduli space of semi-stable bundles at the equivalence class of a stable bundle E is given by

$$T^*_{[E]}M_\sigma \cong H^0(\Sigma_\sigma, \mathrm{End}_0(E) \otimes K).$$

Thinking of $G \in \Omega^0(M, S^2(T))$ as a quadratic function on $T^* = T^*M$, we have that

$$G(v)(\alpha, \alpha) = \int_\Sigma \mathrm{Tr}(\alpha^2)v'$$

where v' is the $(1,0)$-part of v. From this formula it is clear that $G \in H^0(M, S^2(T))$. Furthermore, we observe that $\hat{\nabla}$ agrees with $\hat{\nabla}^t$ along the anti-holomorphic directions $T^{1,0}(\mathcal{T})$.

The particular $u(v) = u_G(v) = u_G$ we are interested in is given by

$$u_G(s) = \frac{1}{2(k+n)}(\Delta_G - 2G\partial F\nabla + ikf_G)s.$$

The leading order term Δ_G is the 2'nd order operator given by

$$\Delta_G : C^\infty(M,\mathcal{L}^k) \xrightarrow{\nabla^{1,0}} C^\infty(M, T^*\otimes\mathcal{L}^k) \xrightarrow{G} C^\infty(M, T\otimes\mathcal{L}^k)$$
$$\xrightarrow{\nabla^{1,0}\otimes 1 + 1\otimes\nabla^{1,0}} C^\infty(M, T^*\otimes T\otimes\mathcal{L}^k) \xrightarrow{\mathrm{Tr}} C^\infty(M,\mathcal{L}^k).$$

The function F is the Ricci potential uniquely determined as the real function with zero average over M, which satisfies the following equation

$$\mathrm{Ric} = 2n\omega + 2i\partial\bar\partial F.$$

We observe that $F \in C^\infty(\mathcal{T}\times M)$, and we define the function $f_G = f_{G(v)} \in C^\infty(M)$ for any $v \in T(\mathcal{T})$ by

$$f_G = -iv[F].$$

Theorem 3 (Axelrod, Della Pietra & Witten; Hitchin)**.** *The expression (1) above defines a unique connection in the bundle $\mathbb{P}(\mathcal{V}_k)$, which is flat.*

Faltings has established this theorem in the case of general semi-simple Lie groups $\mathfrak{G}$, (see [Fal]).

We remark about genus 2, that [ADW] covers this case, but [H] excludes this case, however, the work of Van Geemen and De Jong [vGdJ] extends Hitchin's approach to the genus 2 case.

As discussed in the introduction, we see by Laszlo's theorem that this particular connection is the relevant one to study.

4. Toeplitz operators on compact Kähler manifolds

In this section (M^{2d},ω) will denote a compact Kähler manifold on which we have a holomorphic line bundle $\mathcal{L}$ admitting a compatible Hermitian connection, whose curvature is the symplectic form ω. For each $f \in C^\infty(M)$ consider the associated Toeplitz operator $T_f^{(k)}$ given as the composition of the multiplication operator $M_f : H^0(M,\mathcal{L}^k) \to C^\infty(M,\mathcal{L}^k)$ with the orthogonal projection $\pi : C^\infty(M,\mathcal{L}^k) \to H^0(M,\mathcal{L}^k)$. Sometimes we will suppress the superscript (k) and just write $T_f = T_f^{(k)}$. We remark that we can also consider T_f as an operator from $L_2(M,\mathcal{L}^k)$ to $H^0(M,\mathcal{L}^k)$.

Let us here give an explicit formula for π: On $L_2(M,\mathcal{L}^k)$ we have the defining inner product:

$$\langle s_1, s_2\rangle = \frac{1}{d!}\int_M (s_1,s_2)\omega^d$$

where $s_i \in L_2(M,\mathcal{L}^k)$ and $(\cdot,\cdot)$ is the fiberwise Hermitian structure in $\mathcal{L}^k$. Let now $h_{ij} = \langle s_i, s_j\rangle$, where s_i is a basis of $H^0(M,\mathcal{L}^k)$. Let h_{ij}^{-1}

be the inverse matrix of h_{ij}. Then

$$\pi(s) = \sum_{i,j} \langle s, s_i \rangle h_{ij}^{-1} s_j.$$

This formula will be useful, when we have to compute the derivative of π along a one-parameter curve of complex structures.

Suppose we have a smooth section $X \in C^\infty(TM)$ of the holomorphic tangent bundle of M. We then claim that the operator $\pi\nabla_X$ is a zero-order Toeplitz operator. Suppose $s_1 \in C^\infty(M, \mathcal{L}^k)$ and $s_2 \in H^0(M, \mathcal{L}^k)$, then we have that

$$X(s_1, s_2) = (\nabla_X s_1, s_2).$$

Now, calculating the Lie derivative along X of $(s_1, s_2)\frac{\omega^d}{d!}$ and using the above, one obtains after integration that

$$\langle \nabla_X s_1, s_2 \rangle = -\langle \Lambda d(i_X \omega) s_1, s_2 \rangle,$$

where Λ denotes contraction with ω. Thus

$$\pi\nabla_X = T^{(k)}_{f_X}, \tag{2}$$

as operators from $C^\infty(M, \mathcal{L}^k)$ to $H^0(M, \mathcal{L}^k)$, where $f_X = -\Lambda d(i_X\omega)$. Iterating this, we find for all $X_1, X_2 \in C^\infty(TM)$ that

$$\pi\nabla_{X_1}\nabla_{X_2} = T^{(k)}_{(X_2(f_{X_1}) + f_{X_2} f_{X_1})} \tag{3}$$

again as operators from $C^\infty(M, \mathcal{L}^k)$ to $H^0(M, \mathcal{L}^k)$.

We need the following theorems on Toeplitz operators. The first is due to Bordemann, Meinrenken and Schlichenmaier (see [BMS]).

Theorem 4 (Bordemann, Meinrenken and Schlichenmaier). *For any $f \in C^\infty(M)$ we have that*

$$\lim_{k \to \infty} \|T^{(k)}_f\| = \sup_{x \in M} |f(x)|.$$

Since the association of the sequence of Toeplitz operators T^k_f, $k \in \mathbb{Z}_+$ is linear in f, we see from this theorem, that this association is faithful.

Theorem 5 (Schlichenmaier). *For any pair of smooth functions $f_1, f_2 \in C^\infty(M)$, we have an asymptotic expansion*

$$T^{(k)}_{f_1} T^{(k)}_{f_2} \sim \sum_{l=0}^{\infty} T^{(k)}_{c_l(f_1, f_2)} k^{-l},$$

where $c_l(f_1,f_2) \in C^\infty(M)$ are uniquely determined since $\sim$ means the following: For all $L \in \mathbb{Z}_+$ we have that

$$\|T^{(k)}_{f_1} T^{(k)}_{f_2} - \sum_{l=0}^{L} T^{(k)}_{c_l(f_1,f_2)} k^{-l}\| = O(k^{-(L+1)}).$$

Moreover, $c_0(f_1,f_2) = f_1 f_2$.

This theorem is proved in [Sch], where it is also proved that the formal generating series for the $c_l(f_1,f_2)$'s gives a formal deformation quantization of the Poisson structure on M induced from ω. By examining the proof in [Sch] of this theorem, one observes that for families of functions, the estimates in theorem 5 are uniform over compact parameter spaces.

Theorem 6 (Karabegov & Schlichenmaier)**.** *For any pair of smooth functions $f_1, f_2 \in C^\infty(M)$ the bilinear maps $c_l(f_1,f_2) \in C^\infty(M)$, $l \in \mathbb{Z}_+$ are given by bi-differential operator of order at most (l,l), in particular $c_l(f_1,f_2)(p)$ only depends on the l'th jet of f_1 and f_2 at p for each $p \in M$.*

This theorem is proved in [KS], where furthermore the resulting $*$-product mentioned above is identified in terms of Karabegov's classification of $*$-products with separation of variables on Kähler manifolds.

5. Toeplitz operators on moduli space and the projective flat connection

Let $f \in C^\infty_c(M')$ be a smooth function on the moduli space with compact support on M'. We consider $T^{(k)}_f$ as a section of the endomorphism bundle $\mathrm{End}(\mathcal{V}_k)$. The flat connection $\hat{\nabla}$ in the projective bundle $\mathbb{P}(\mathcal{V}_k)$, induces a flat connection $\hat{\nabla}^{\mathrm{e}}$ in the endomorphism bundle $\mathrm{End}(\mathcal{V}_k)$. We shall now establish that the sections $T^{(k)}_f$ are in a certain sense asymptotically flat by proving the following theorem.

Theorem 7. *Let σ_0 and σ_1 be two points in Teichmüller space and P_{σ_0,σ_1} be the parallel transport in the flat bundle* $\mathrm{End}(\mathcal{V}_k)$ *from σ_0 to σ_1. Then*

$$\|P_{\sigma_0,\sigma_1} T^{(k)}_{f,\sigma_0} - T^{(k)}_{f,\sigma_1}\| = O(k^{-1}),$$

where $\|\cdot\|$ is the operator norm on $H^0(M,\mathcal{L}^k)$.

In order to prove this theorem, one needs of course estimates on $\|\hat{\nabla}^e_{\sigma_t'} T^{(k)}_{f,\sigma_t}\|$ along a smooth one-parameter family of complex structures σ_t ($t \in J = [0,1]$) from σ_0 to σ_1. We will see (Proposition 2 below) that $\hat{\nabla}^{\mathrm{e}}$ is asymptotically well related to the tensor product norm on $(H^0(M,\mathcal{L}^k))^* \otimes H^0(M,\mathcal{L}^k)$ induced from (a slightly twisted version of) the L_2-norm on $H^0(M,\mathcal{L}^k)$. However, the operator norm and the tensor product norm on $H^0(M,\mathcal{L}^k) \otimes (H^0(M,\mathcal{L}^k))^*$ are related by a factor equal to the square root of $\dim H^0(M,\mathcal{L}^k)$, which is a polynomial in k of degree $d = (n^2-1)(g-1)$ (see (5) below). Hence we really need estimates on the covariant derivative, which decays faster than $k^{-d/2}$ to establish Theorem 7. In order to obtain that, one needs to perturb the function f by adding on sufficiently many terms of the form $(h_l)_t k^{-l}$ ($l = 1, ..., m$), where $(h_l)_t \in C_c^\infty(M')$, $t \in J$, to obtain a smooth one-parameter family of functions $(f_m)_t = f + \sum_{l=1}^m (h_l)_t k^{-l}$ for an $m > (n^2-1)(g-1)/2 - 1$ which satisfies that $\| \hat{\nabla}^e_{\sigma_t'} T^{(k)}_{(f_m)_t,\sigma_t} \|$ is $O(k^{-m-1})$. This is the content of Proposition 1 below.

However, in order to choose these functions h_l correctly, we need to do a few elementary calculations.

Along any smooth one-parameter family of complex structures σ_t (index by J), consider a basis of covariant constant sections $s_i = (s_i)_t$, $i = 1, \dots, N$ of $\mathcal{V}_k$ over σ_t:

$$s_i' = u_G(s_i), \qquad i = 1, \dots N.$$

The projection π, as we saw above, is then given by

$$\pi(s) = \sum_{i,j} \langle s, s_i \rangle h_{ij}^{-1} s_j.$$

Then

$$\begin{aligned}\pi'(s) \quad &= \quad \sum_{i,j} \langle s, s_i' \rangle h_{ij}^{-1} s_j \\ &\qquad + \sum_{i,j} \langle s, s_i \rangle (h_{ij}^{-1})' s_j \\ &\qquad + \sum_{i,j} \langle s, s_i \rangle h_{ij}^{-1} s_j'.\end{aligned}$$

An easy computation gives that

$$(h_{ij}^{-1})' = -\sum_{l,m} h_{il}^{-1} (\langle s_l', s_m \rangle + \langle s_l, s_m' \rangle) h_{mj}^{-1},$$

so

$$\begin{aligned}\pi\pi'(s) &= \sum_{i,j}\langle u_G^* s, s_i\rangle h_{ij}^{-1} s_j \\ &\quad - \sum_{i,l,m,j}\langle s, s_i\rangle h_{il}^{-1}\langle s_l, s'_m\rangle h_{mj}^{-1} s_j \\ &= \pi u_G^* - \sum_{m,j}\langle \pi s, s'_m\rangle h_{mj}^{-1} s_j.\end{aligned}$$

Hence we conclude that

Lemma 1.

$$\pi\pi' = \pi u_G^* - \pi u_G^* \pi.$$

Let m be a non-negative integer and let

$$(f_m)_t = \sum_{i=0}^{m} (h_i)_t k^{-i},$$

where the h_i's are arbitrary smooth one-parameter family of functions $C_c^\infty(M')$ indexed by J , except, we require that $h_0 = f$. We will from now mostly suppress the subscript t on all quantities. We have that $T_{f_m}^{(k)}$ is a section of $\operatorname{End}(\mathcal{V}_k)$ over the curve σ. Let

$$A_m = \hat{\nabla}^e_{\sigma'}(T_{f_m}) = (T_{f_m})' - [u_G, T_{f_m}] = \pi f'_m + \pi' f_m - [u_G, \pi f_m].$$

Since $\hat{\nabla}^e_{\sigma'}(T_{f_m})$ is a section of $\operatorname{End}(\mathcal{V}_k)$, we have of course that

$$\pi A_m \pi = A_m \pi : H^0(M_{\sigma_t}, \mathcal{L}^k) \to H^0(M_{\sigma_t}, \mathcal{L}^k).$$

Proposition 1. *Given f and a non-negative integer, there exists unique smooth one-parameter families of functions $(h_i)_t \in C_c^\infty(M)$, $i = 1, \dots, m$ and $t \in J$ such that*

$$\sup_{t\in J} \|A_m \pi\| = O(k^{-m-1}),$$

and $(h_i)_0 = 0$, $i = 1, \dots, m$.

Proof. For any choice of h_i's, $i = 1, \dots, m$, we compute that

$$\begin{aligned}\pi A_m \pi = &\sum_{i=1}^{m} \pi h'_i \pi k^{-i} \\ &+ \sum_{i=0}^{m} (\pi u_G^* h_i \pi - \pi u_G^* \pi h_i \pi) k^{-i} \\ &- \sum_{i=0}^{m} (\pi u_G \pi h_i \pi - \pi h_i u_G \pi) k^{-i}.\end{aligned}$$

Pulling $\mathcal{L}^k$ back to the desingularization (which we also denote M_σ) of M_σ, if M_σ is singular and using (2) and (3) we define the function H_G, as follows

$$\pi(\Delta_G - 2G\partial F\nabla) = \pi H_G.$$

We also define $H_i \in C_c^\infty(M')$ which depends on h_i and G by

$$\pi h_i(\Delta_G - 2G\partial F\nabla) = \pi H_i.$$

Then we have that

$$\begin{aligned}\pi A_m \pi = &\sum_{i=1}^m \pi h_i' \pi k^{-i}\\ &- \sum_{i=0}^m \frac{1}{2(k+n)}(\pi \overline{H}_G \pi h_i \pi - \pi \overline{H}_G h_i \pi) k^{-i}\\ &- \sum_{i=0}^m \frac{1}{2(k+n)}(\pi H_G \pi h_i \pi - \pi H_i \pi) k^{-i}\\ &+ \sum_{i=0}^m \frac{k}{2(k+n)} i(\pi \overline{f}_G \pi h_i \pi - \pi \overline{f}_G h_i \pi) k^{-i}\\ &+ \sum_{i=0}^m \frac{k}{2(k+n)} i(\pi f_G \pi h_i \pi - \pi h_i f_G \pi) k^{-i}.\end{aligned}$$

We will now argue that we can for arbitrary integer m uniquely determine the functions h_i with the properties stated in Proposition 1. In order to do that, replace each product of Toeplitz operators on the left hand side of the above expression by their asymptotic expansion as given by Theorem 6. In the last two terms we notice after this substitution, that the terms in the bracket is now proportional to k^{-1}, since the leading order term in the expansion of the product of two Toeplitz operators, is just the Toeplitz operator associated to the product of the two functions.

Suppose now that we have uniquely determined $(h_i)_t \in C_c^\infty(M)$, $i = 1, \dots, m-1$ and $t \in J$ such that

$$\sup_{t\in J} \|A_{m-1}\pi\| = O(k^{-m}),$$

and $(h_i)_0 = 0$, $i = 1, \dots, m-1$. By considering an arbitrary $h_m \in C_c^\infty(M')$ and collecting up terms, we find that the total coefficient of k^{-m} of the right hand side is the Toeplitz operator associated to h_m' minus an expression in h_i, H_i, $i = 1, \dots, m-1$ and H_G. By theorem 4 we now see that A_m satisfies the required norm-estimate if and only if h_m' equals this expression in h_i, H_i, $i = 1, \dots, m-1$ and H_G, which is a smooth one-parameter family of functions in $C_c^\infty(M')$ by the locality of the c_l's as provided by Theorem 6. E.g. considering the coefficient of k^{-1}, we find that h_1 has to satisfy that

$$h_1' = -\frac{1}{2}(H_G h_0 - H_0 - c_1(\bar{f}_G, h_0) - c_1(f_G, h_0)),$$

where the right hand side is seen to be a smooth one-parameter family of functions in $C_c^\infty(M')$, since $h_0 = f \in C_c^\infty(M')$.

By simply integrating the resulting expression for h_m' from 0 to t, we get the unique solution $(h_m)_t \in C_c^\infty(M')$, which satisfies that $(h_m)_0 = 0$. By its very construction, this h_m and the h_i, $i = 1, \dots, m-1$ gives an f_m, whose associated A_m satisfies the required norm estimate.

□

Consider now the following Hermitian structure on $\mathcal{H}_k$:

$$\langle s_1, s_2 \rangle_F = \frac{1}{d!} \int_M (s_1, s_2) e^{\frac{1}{2}F} \omega^d \tag{4}$$

We claim that $\langle \cdot, \cdot \rangle_F$ and $\langle \cdot, \cdot \rangle$ are equivalent uniformly in k on $\mathcal{V}_k$ over any compact subset K of $\mathcal{T}$.

We clearly have that

$$|s|_F = |T^{(k)}_{e^{\frac{1}{4}F}} s| \leq \|T^{(k)}_{e^{\frac{1}{4}F}}\| \; |s|,$$

so by Theorem 4 we see that there exists a constant C (depending on K) such that

$$|s|_F \leq C|s|,$$

for all k. Conversely, we have that

$$\begin{aligned} |s|^2 &= \langle \pi e^{-\frac{1}{4}F} e^{\frac{1}{4}F} \pi s, s \rangle \\ &\leq |\langle (\pi e^{-\frac{1}{4}F} e^{\frac{1}{4}F} \pi - \pi e^{-\frac{1}{4}F} \pi e^{\frac{1}{4}F} \pi) s, s \rangle| \\ &\quad + |\langle \pi e^{-\frac{1}{4}F} \pi e^{\frac{1}{4}F} \pi s, s \rangle| \\ &\leq \|\pi e^{-\frac{1}{4}F} e^{\frac{1}{4}F} \pi - \pi e^{-\frac{1}{4}F} \pi e^{\frac{1}{4}F} \pi\| \; |s|^2 \\ &\quad + \|\pi e^{-\frac{1}{4}F} \pi\| \; |\pi e^{\frac{1}{4}F} \pi s| \; |s|. \end{aligned}$$

By Theorem 4 and 5 we see there exists constants C' and C'' (again depending on K) such that

$$|s| \leq \frac{C'}{k}|s| + C''|s|_F.$$

But then we have for all sufficiently large k that

$$|s| \leq 2C''|s|_F.$$

Hence we have established the claimed equivalence.

Along any smooth one-parameter family of complex structures σ_t, we have that

$$\frac{d}{dt}\langle s_1, s_2\rangle_F = \langle \hat{\nabla}^t_{\sigma'_t} s_1, s_2\rangle_F + \langle s_1, \hat{\nabla}^t_{\sigma'_t} s_2\rangle_F + \langle \frac{1}{2}\frac{\partial F}{\partial t} s_1, s_2\rangle_F.$$

So, if we let

$$E(s) = \frac{d}{dt}|s|^2_F - \langle \hat{\nabla}_{\sigma'_t} s, s\rangle_F - \langle s, \hat{\nabla}_{\sigma'_t} s\rangle_F$$

we have for all sections s of $\mathcal{V}_k$ that

$$\begin{aligned} E(s) = \frac{1}{2(k+n)}(&\langle \pi e^{\frac{1}{2}F}(\Delta_G - 2G\partial F\nabla - inf_G)s, s\rangle \\ &+ \langle s, \pi e^{\frac{1}{2}F}(\Delta_G - 2G\partial F\nabla - inf_G)s\rangle) \end{aligned}$$

Hence by combining Theorem 4, (2) and (3) we have proved that

Proposition 2. *The Hermitian structure (4) is asymptotically flat with respect to the connections $\hat{\nabla}$, i.e. for any compact subset K of $\mathcal{T}$, there exists a constant C such that for all sections s of $\mathcal{V}_k$ over K, we have that*

$$|E(s)| \leq \frac{C}{k+n}|s|^2_F$$

over K.

We note that this proposition implies the same proposition for sections of the $\mathrm{End}(\mathcal{V}_k)$ with respect to the induced inner product on $\mathrm{End}(\mathcal{V}_k) = \mathcal{V}_k^* \otimes \mathcal{V}_k$, which we also denote $\langle \cdot, \cdot\rangle_F$. We denote the analogous quantity of E for the endomorphism bundle by E_e.

Proof of Theorem 7. Let σ_t, $t \in J$ be a smooth one-parameter family of complex structures such that σ_t is a curve in $\mathcal{T}$ between the two points in question. Choose an $m > d/2$ and f_m as provided by Proposition 1. Define $n_k : J \to \overline{\mathbb{R}}_+$ by

$$n_k(t) = |P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}|^2_F$$

Then

$$\begin{aligned} \frac{dn_k}{dt} &= \langle \hat{\nabla}^e_{\sigma'_t}(P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}), P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}\rangle_F \\ &\quad + \langle P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}, \hat{\nabla}^e_{\sigma'_t}(P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t})\rangle_F \\ &\quad + E_e(P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}) \\ &= -\langle \hat{\nabla}^e_{\sigma'_t} T^{(k)}_{f_m,\sigma_t}, P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}\rangle_F \\ &\quad - \langle P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}, \hat{\nabla}^e_{\sigma'_t} T^{(k)}_{f_m,\sigma_t}\rangle_F \\ &\quad + E_e(P_{\sigma_0,\sigma_t} T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}). \end{aligned}$$

Recall that we have the following inequalities between the tensor product norm and the operator norm on $\mathcal{V}_k$:

$$\|\cdot\| \leq |\cdot| \leq \sqrt{P_{g,n}(k)}\|\cdot\|, \tag{5}$$

where $P_{g,n}(k)$ is the rank of $\mathcal{V}_k$ given by the Verlinde formula. By the Lefschetz-Rieman-Roch Theorem this is a polynomial in k of degree $d/2$. Since $|\cdot|$ and $|\cdot|_F$ are equivalent uniformly in k, we get that there exists a constant C such that

$$\begin{aligned}|\frac{dn_k}{dt}| &\leq 2|\,\hat{\nabla}^e_{\sigma_t'} T^{(k)}_{f_m,\sigma_t}|_F |P_{\sigma_0,\sigma_t}T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t}|_F \\ &\quad + |E_e(P_{\sigma_0,\sigma_t}T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t})| \\ &\leq 2C\sqrt{P_{g,n}(k)}\|\,\hat{\nabla}_{\frac{\partial}{\partial t}} T^{(k)}_{f_m,\sigma_t}\| n_k^{1/2} \\ &\quad + |E_e(P_{\sigma_0,\sigma_t}T^{(k)}_{f_m,\sigma_0} - T^{(k)}_{f_m,\sigma_t})|.\end{aligned}$$

Consequently we can apply Proposition 1 and 2 to obtain there exists a constant C' such that

$$|\frac{dn_k}{dt}| \leq \frac{C'}{k}(n_k^{1/2} + n_k).$$

This estimate implies that

$$n_k(t) \leq (\exp(\frac{C't}{2k}) - 1)^2.$$

Which therefore implies that

$$\|P_{\sigma_0,\sigma_1}T^{(k)}_{(f_m)_0,\sigma_0} - T^{(k)}_{(f_m)_1,\sigma_1}\| \leq C'' n_k(1)^{1/2},$$

for some constant C''. The theorem then follows from this estimate, since $(f_m)_0 = f$ and $\|T^{(k)}_{(f_m)_1,\sigma_1} - T^{(k)}_{f,\sigma_1}\| = O(k^{-1})$.

□

6. Asymptotic faithfulness

Recall that the flat connection on the bundle $\mathbb{P}(\mathcal{V}_k)$ gives the projective representation of the mapping class group

$$\rho_k : \Gamma \to \mathrm{Aut}(\mathbb{P}(V_k)),$$

where $\mathbb{P}(V_k)$ = covariant constant sections of $\mathbb{P}(\mathcal{V}_k)$ over Teichmüller space.

Theorem 8. *For any $\phi \in \Gamma$, we have that*

$$\phi \in \bigcap_{k=1}^{\infty} \ker \rho_k$$

if and only if ϕ induces the identity on M.

Proof. Suppose we have a $\phi \in$, . Choose a σ in $\mathcal{T}$. If $f \in C_c^\infty(M')$ then $f \circ \phi \in C_c^\infty(M')$ and we get the following commutative diagram

$$\begin{tikzcd}
H^0(M_\sigma, \mathcal{L}^k) \arrow[r, "\phi^*"] \arrow[d, "T^{(k)}_{f,\sigma}"'] & H^0(M_{\phi(\sigma)}, \mathcal{L}^k) \arrow[r, "P_{\phi(\sigma),\sigma}"] \arrow[d, "T^{(k)}_{f\circ\phi,\phi(\sigma)}"'] & H^0(M_\sigma, \mathcal{L}^k) \arrow[d, "P_{\phi(\sigma),\sigma} T^{(k)}_{f\circ\phi,\phi(\sigma)}"] \\
H^0(M_\sigma, \mathcal{L}^k) \arrow[r, "\phi^*"] & H^0(M_{\phi(\sigma)}, \mathcal{L}^k) \arrow[r, "P_{\phi(\sigma),\sigma}"] & H^0(M_\sigma, \mathcal{L}^k),
\end{tikzcd}$$

where $P_{\phi(\sigma),\sigma} : H^0(M_{\phi(\sigma)}, \mathcal{L}^k) \to H^0(M_\sigma, \mathcal{L}^k)$ on the horizontal arrows refer to parallel transport in the Verlinde bundle itself, whereas it refers to the parallel transport in the endomorphism bundle $\operatorname{End}(\mathcal{V}_k)$ on the last vertical arrow. Suppose now $\phi \in \bigcap_{k=1}^\infty \ker \rho_k$, then $P_{\phi(\sigma),\sigma} \circ \phi^* = \rho_k(\phi) \in \mathbb{C}\,\mathrm{Id}$ and we get that $T^{(k)}_{f,\sigma} = P_{\phi(\sigma),\sigma} T^{(k)}_{f\circ\phi,\phi(\sigma)}$. By theorem 7 we get that

$$\begin{aligned}
\lim_{k\to\infty} \|T^{(k)}_{f - f\circ\phi,\sigma}\| &= \lim_{k\to\infty} \|T^{(k)}_{f,\sigma} - T^{(k)}_{f\circ\phi,\sigma}\| \\
&= \lim_{k\to\infty} \|P_{\phi(\sigma),\sigma} T^{(k)}_{f\circ\phi,\phi(\sigma)} - T^{(k)}_{f\circ\phi,\sigma}\| = 0.
\end{aligned}$$

By Bordemann, Meinrenken and Schlichenmaier's theorem 4, we must have that $f = f \circ \phi$. Since this holds for any $f \in C_c^\infty(M')$, we must have that ϕ acts by the identity on M.

□

Proof of Theorem 1. Our main theorem 1 now follows directly from Theorem 8, since it is known that the only element of , , which acts by the identity on the $SU(2)$ moduli space M is the identity, if $g > 2$, and if $g = 2$, it is contained in the sub-group generated by the hyper-elliptic involution. A way to see this using the moduli space of flat $SL(2,\mathbb{C})$-connections goes as follows: For $n = 2$ we have that M is a real slice in the $SL(2,\mathbb{C})$ moduli space $\mathcal{M}$. Hence if ϕ acts by the identity on M, it will also act by the identity on an open neighbourhood of M in $\mathcal{M}$, since it acts holomorphicly on $\mathcal{M}$. But since $\mathcal{M}$ is connected, ϕ must act by the identity on the entire $SL(2,\mathbb{C})$-moduli space $\mathcal{M}$. Now $\mathcal{T}$ Teichmüller space is also included in $\mathcal{M}$, hence we get that ϕ acts by the identity on $\mathcal{T}$. But then the statement about ϕ follows by classical theory of how , acts on $\mathcal{T}$.

For $n > 2$, we just have to check that the hyper-elliptic involution act non trivial for all $n > 2$, since the $SU(2)$-moduli space embeds into the $SU(n)$-moduli space. However, it is a simple exercise in the use of the Lefschetz fixed point theorem, to check that the dimension of the fixed point set of the hyper-elliptic involution in genus 2 in the

$SU(n)$-moduli space space is strictly less than the dimension of that moduli space itself if and only if $n > 2$.

□

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