Graduate Course on

Applied Analysis

University of Copenhagen, Autumn 2000

Lectures by

Professor Michael Vogelius, Rutgers University

Professor Michael Vogelius from Rutgers University, who is currently visiting the University of Copenhagen (partially funded by the Danish Research Training Council and MaPhySto) will give a graduate course on applied analysis at the University of Copenhagen and the Technical University of Denmark.

Course Title: Topics in Applied Analysis

Course Contents: For further description please see the back side of this announcement.

Course Schedule: The course will be concentrated during the months of November 2000 and January 2001 with 15 hours of lectures each month (for a total of 30 hours of lectures). In November there will be 6 two-hour lectures (2 two-hour lectures per week in week nos. 45, 46 and 47) and one three-hour lecture (in week no. 48). The exact dates and times will be decided in accordance with the preferences of the participants. In January there will be 3 two-hour lectures during week no. 2, and 3 three-hour lectures (on the 15th, the 18th and the 22nd). Two different (but related) topics will be covered. It is possible to follow only the lectures on one topic.

The lectures on the 15th, 18th, and 22nd of January 2001 will take place at the Technical University. All other lectures will be at the University of Copenhagen.

Credits: Students at the University of Copenhagen who wish to obtain credit for the course are required to attend both parts, and they will then be credited “2 punkter”.

Prerequisites: The course should be accessible to mathematics, physics as well as advanced engineering students with some elementary knowledge of Partial Differential Equations. The speed as well as the particular emphasis may to some extent be adjusted to the interests of the audience.

Registration: by e-mail to one of the contact persons:

- J.P. Solovej, `solovej@math.ku.dk`
- M.S. Vogelius, `vogelius@math.ku.dk`

by October 20th. The final schedule will then, in consultation with the participants, be decided during the following week.

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Course Description:

November 2000 (first part):

Homogenization and effective media theories

The main goal of this part of the course is to find the macroscopic (effective) behaviour of models of continuum mechanics with microstructure. These models involve partial differential equations (PDE's), for instance the equations of elasticity or simplified scalar versions, with highly oscillatory coefficients. The resulting (effective) macroscopic models have no microstructure (in the simplest case they involve constant coefficient PDE's) but they are constructed in such a way that the corresponding solutions are the appropriate (weak) limits of the solutions to the models with microstructure (i.e. such that they reproduce the correct "average" fields).
We will begin by studying models with periodic (or "locally periodic") microstructure, but then we will proceed to rather arbitrary microstructures and give a fairly elementary introduction to the theory of H-convergence and compensated compactness (primarily due to Murat and Tartar). A central topic will be bounds for effective media (f.ex. bounds for the eigenvalues of the tensors associated with the constant coefficient PDE's). This will for example cover the simplest examples of geometry independent bounds (sometimes referred to as Hashin Shtrikman bounds) and their rigorous derivation. In this context the course will also briefly describe various connections to questions of (generalized) optimal design. Time permitting the course will then return to periodic microstructures to provide a study of the effective modelling of boundary (and interface) layers.

January 2001 (second part):

Inverse problems for elliptic differential operators

One often seeks to find solutions to models of continuum mechanics (governed by partial differential equations) assuming knowledge of the material composition. Looking at it from the opposite direction one may of course view "nature" as a perfect solver of these equations, and then use accessible measurements of "nature's" solution in order to gain information about the material composition.

A simple, but very fundamental question motivating this course is the following:

"To what extent is the coefficient $a(x)$ in the equation $\mathrm{div}(a(x)\ \mathrm{grad}\ u) = 0$ determined from just boundary information about solutions u?"

Variants of this question can (and will) be asked for the Maxwell Equations, the Equations of Elasticity, the Schrödinger Equation .... etc. Practical applications are found in medical imaging (impedance computed tomography and other forms of tomography) in detection of flaws in metal components (by a method referred to as the eddy current technique).

Specifically the course will

(1) establish that a (sufficiently smooth) isotropic coefficient $a(x)$ is uniquely determined by the Cauchy (boundary) data of all possible solutions to $\mathrm{div}\ (a(x)\mathrm{grad}\ u) = 0$ (in dimension greater than 2),

(2) show that there is not a similar uniqueness theorem for anisotropic coeffients (and discuss what one may determine),

(3) consider the added complications present in 2 dimensions,

(4) prove that a finite number of Cauchy data (typically one or two) makes it possible to determine coefficients $a(x)$ that correspond to a known background medium with a collection of (unknown) cracks or small imperfections,

(5) discuss numerical algorithms.